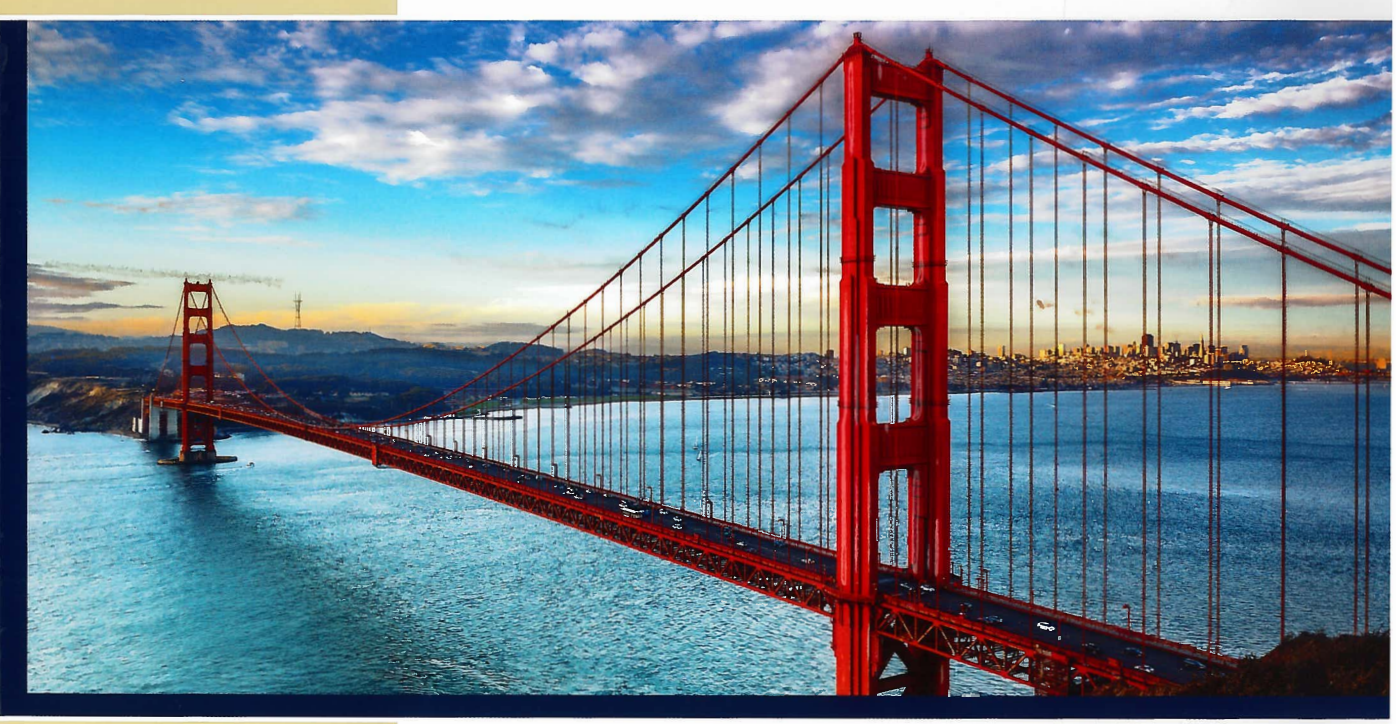
SCIENTIFIC SECTION

Pure Mathematics

By a group of supervisors

Interactive exams using QR CODE



EXERCISES

SECOND TERM 2 SEC. 2024

EL-MOASSER

EL-MOASSER

PURE Mathematics

By a group of supervisors

EXERCISES

SCIENTIFIC SECTION

GPS

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SECOND TERM 2 SEC.

جميع حقوق الطبع والنشر محفوظة

لا يجوز، بأى صورة من الصور، التوصيل (النقل) المباشر أو غير المباشر لأى مما ورد فى هذا الكتاب أو نسخه أو تصويره أو ترجمته أو تحويره أو الاقتباس منه أو تحويله رقميًا أو إتاحته عبر شبكة الإنترنت **إلا بإذن كتابى** مسبق من الناشر.

كما لا يجوز بأى صورة من الصور استخدام العلامة التجارية (EL-MOASSER) المسجلة باسم الناشر

ومَن يخالف ذلك يتعرض للمساءلة القانونية طبقًا لأحكام القانون ٨٢ لسنة ٢٠٠٢ الخاص بحماية الملكية الفكرية.

Preface

Thanks to God who helped us to introduce one of our famous series **"El Moasser"** in mathematics.

We introduce this book to our colleagues. We also introduce it to our students to help them study mathematics.

In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

The Authors

Solve

an interactive exam

for each lesson using the technique of:

QR Code

1

Download an application of

QR code reader

on your smart phone from

Available on the App Store or Get it on Google play

2

Open the application and scan the

QR code

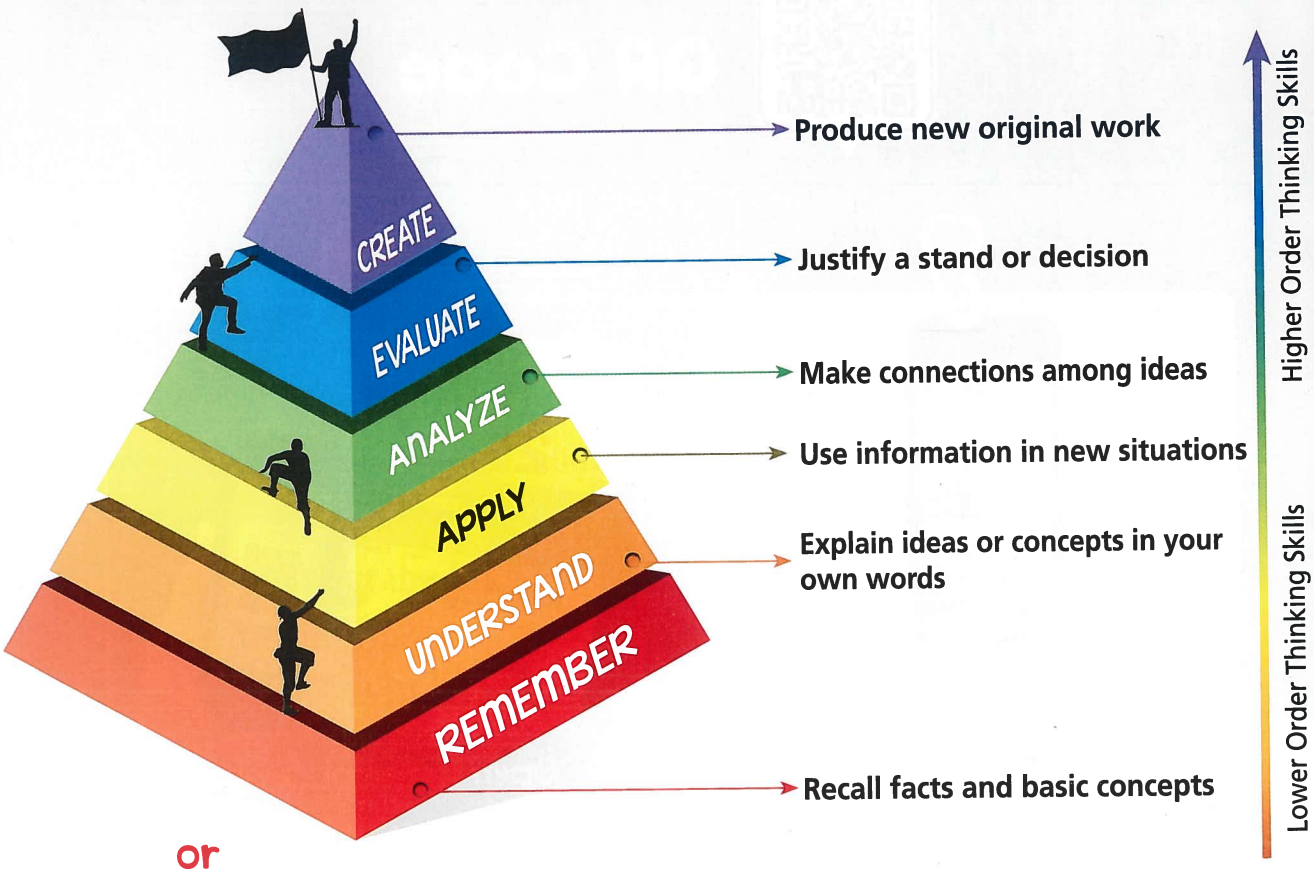
using phone's camera , then start solving directly.

- After finishing the test you can get your marks to evaluate yourself showing you a detailed report for the correct answer.

Bloom's Taxonomy Of Cognitive Levels

Bloom's Taxonomy is an educational classification created by Benjamin Bloom, it is often represented as a pyramid. This taxonomy was revised to include six cognitive levels graded form the lower level to the higher level as follows:

REMEMBER → UNDERSTAND → APPLY → ANALYZE → EVALUATE → CREATE



or

Bloom's Revised Pyramid

Note :

The questions within each exercise are classified according to the levels of Bloom's pyramid and are referred to as follows:

● UNDERSTAND ○ APPLY HIGHER ORDER THINKING SKILLS (HOTS)
(ANALYZE - EVALUATE - CREATE)

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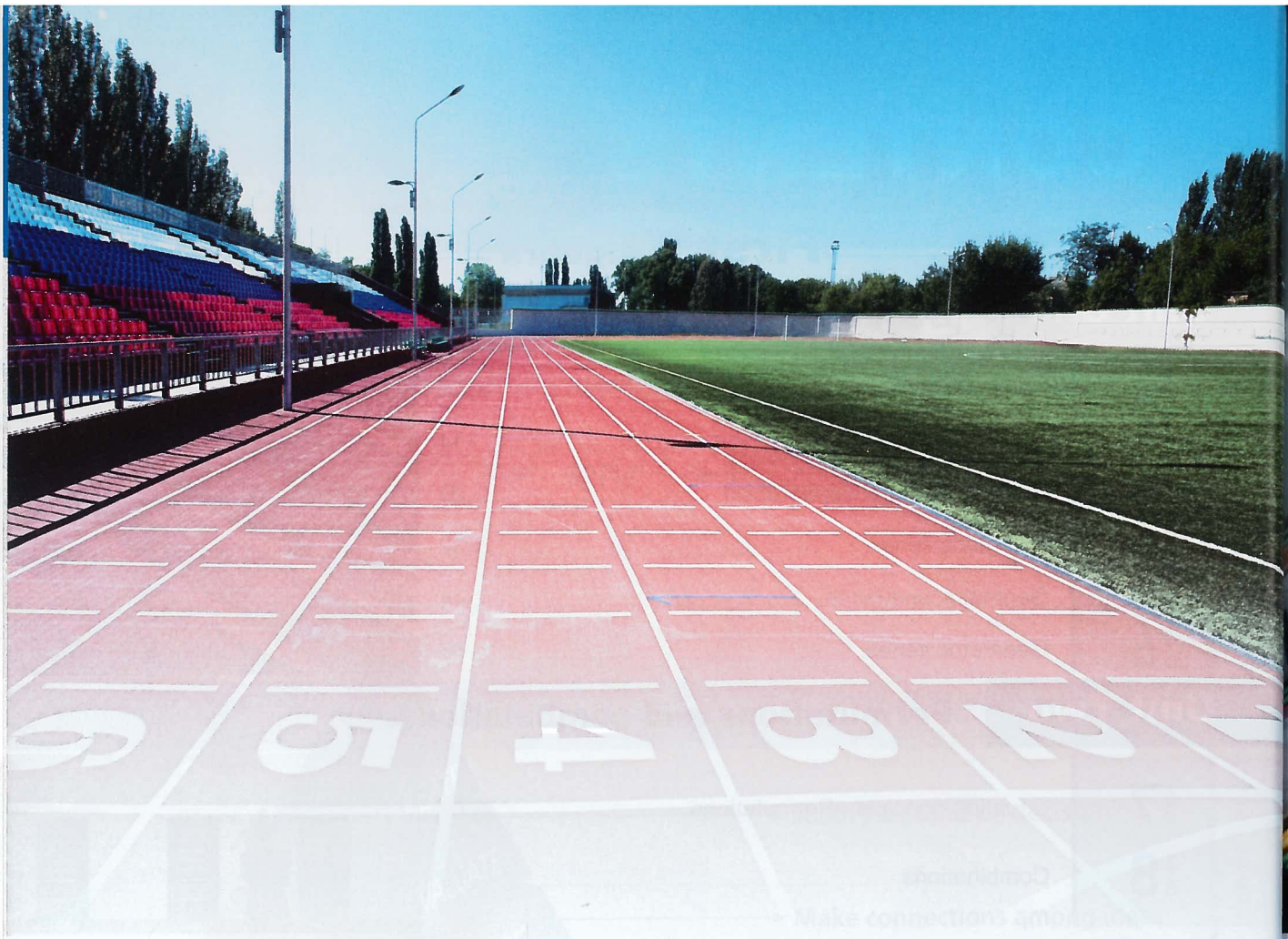


First Algebra

UNIT 1 **Sequences and series.**

UNIT 2 **Permutations and combinations.**

Unit One

Sequences and series.

Exercise 1 Sequences.

Exercise 2 Series and summation notation.

Exercise 3 Arithmetic sequence.

Exercise 4 Arithmetic series.

Exercise 5 Geometric sequence.

Exercise 6 Geometric series.

Scan the QR code to solve an interactive test on each lesson

Exercise 1

Sequences

From the school book | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The sequence is a function whose domain is .....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{Z}$ (d) $\mathbb{Z}^+$

(2) The next term in the sequence (1 , 2 , 4 , 7 , 11 , ...) is .....

(a) 15 (b) 16 (c) 17 (d) 18

(3) The next term in the sequence $\left(\frac{3}{13} , \frac{7}{10} , \frac{11}{7} , \frac{15}{4} , ...\right)$ is .....

(a) $\frac{19}{2}$ (b) 19 (c) $-\frac{23}{2}$ (d) $\frac{16}{3}$

(4) The tenth term of the terms of the sequence (1 , 1 , 2 , 3 , 5 , 8 , 13 , ...) is .....

(a) 29 (b) 34 (c) 55 (d) 89

(5) The term free of x in the sequence $(8 - 6x , 7 - 4x , 6 - 2x , ...)$ is .....

(a) T_7 (b) T_6 (c) T_5 (d) T_4

(6) The sixth term of the sequence (T_n) , where $T_n = \frac{(-1)^n}{2n}$ is .....

(a) 6 (b) $\frac{-1}{12}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

(7) The fourth term of the sequence (T_n) , where $T_n = \frac{\sqrt{n}}{n+1}$ is .....

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{4}{5}$ (d) $\frac{2}{5}$

(8) $T_5 = 7$ in which of the following sequences ?

(a) $(T_n) = (2n + 1)$ (b) $(T_n) = (3n^2 - n)$ (c) $(T_n) = (n + 2)$ (d) $(T_n) = (2^n - 1)$

(9) In the sequence (T_n) , where $T_{n+1} = n\,T_n$, $n \geq 1$, if $T_1 = 1$, then $T_2 =$ .....

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{2}$

(10) The first five terms in the sequence $(T_n) = \big(n\,(n-1)\big)$ are .....

(a) (1 , 2 , 6 , 12 , 20) (b) (0 , 2 , 6 , 10 , 14)

(c) (0 , 2 , 6 , 12 , 20) (d) (1 , 2 , 4 , 7 , 11)

(11) The first five terms in the sequence (T_n) in which $T_{n+1} = \frac{(-1)^n}{3\,T_n}$, $T_1 = 9$ is .....

(a) $\left(9 , \frac{1}{27} , -9 , 27 , 9\right)$ (b) $\left(9 , \frac{1}{9} , -9 , \frac{1}{27} , -27\right)$

(c) $\left(9 , \frac{1}{27} , \frac{-1}{81} , \frac{1}{243} , \frac{-1}{729}\right)$ (d) $\left(9 , \frac{-1}{27} , -9 , \frac{1}{27} , 9\right)$

(12) In the sequence (T_n) , if $T_1 = 2$, $T_{n+1} = T_n - n$ where $n \geq 1$, then the first five terms are .....

(a) 1 , 2 , – 1 , – 4 , – 8 (b) 2 , – 1 , 1 , 3 , – 8

(c) 2 , 1 , – 1 , – 4 , – 8 (d) 2 , 1 , – 1 , – 3 , – 8

(13) The first five terms of the sequence in which $T_1 = 1$, $T_2 = 2$, $T_n = T_{n-1} + T_{n-2}$ for every $n > 2$ are .....

(a) (1 , 2 , 3 , 4 , 5) (b) (1 , 2 , 4 , 8 , 16)

(c) (1 , 2 , 3 , 5 , 8) (d) (1 , 2 , 3 , 6 , 12)

(14) Given that (T_n) is a sequence in which $T_n = T_1 + T_2 + T_3 + ... + T_{n-1}$ where $n > 2$, then $T_1 + T_2 + T_3 + T_4 + T_5 - T_6 =$ .....

(a) zero (b) 32 (c) 64 (d) 72

(15) If the general term of a sequence (T_n) is $T_n = 3$, then the first five terms in it are .....

(a) (3 , 4 , 5 , 6 , 7) (b) (1 , 2 , 3 , 4 , 5)

(c) (3 , 3 , 3 , 3 , 3) (d) (4 , 5 , 6 , 7 , 8)

(16) The sequence is decreasing if T_{n+1} ..... T_n for every $n \geq 1$

(a) > (b) < (c) = (d) ≥

(17) It is said that the sequence (T_n) is increasing for all values of $n \in \mathbb{Z}^+$ if .....

(a) $\frac{T_n}{T_{n+1}} > 1$ (b) $T_n = T_{n+1}$ (c) $T_{n+1} > T_n$ (d) $-1 < \frac{T_n}{T_{n+1}} < 0$

(18) The sequence whose rule is $T_n = \frac{(-1)^n}{3n-2}$ is called ..... sequence.

(a) an increasing (b) a decreasing (c) an alternating (d) a constant

(19) The sequence whose n^{th} term is $T_n = \frac{2}{n} - 1$, where $n \in \mathbb{Z}^+$ represents .....

(a) an increasing sequence. (b) a decreasing sequence.

(c) a constant sequence. (d) an alternating sequence.

(20) The general term of a sequence is $T_n = \left(\frac{1}{3}\right)^{n+1}$ then the sequence is .....

(a) increasing. (b) decreasing. (c) alternating. (d) finite.

(21) The sequence $(T_n) = \left((-1)^n (n^2 + 1)\right)$ is .....

(a) increasing. (b) decreasing. (c) alternating. (d) constant.

(22) Which of the following sequences (T_n) is decreasing where $n \in \mathbb{Z}^+$?

(a) $(T_n) = (-n^2)$ (b) $(T_n) = (n)$ (c) $(T_n) = (n^3)$ (d) $(T_n) = (n + 1)$

(23) Which of the following represents a general term of a decreasing sequence ?

(a) n (b) n^2 (c) $n^2 - 5$ (d) $\frac{n+1}{n}$

(24) The sequence $(1, 4, 7, 11, ...)$ is .....

(a) decreasing. (b) alternating. (c) finite. (d) infinite.

(25) The sequence $(3, 5, 7, 9, ..., 21)$ is .....

(a) decreasing. (b) alternating. (c) finite. (d) infinite.

(26) The sequence $(T_n) = (n^2 - 1), n \in \mathbb{Z}^+$ is .....

(a) decreasing. (b) alternating. (c) finite. (d) infinite.

(27) The sequence (T_n) where $T_n = \frac{3}{n} + 1, n \in \{1, 2, 3, 4, 5\}$ is .....

(a) increasing. (b) alternating. (c) finite. (d) infinite.

(28) The n^{th} term of the sequence $(1, 4, 9, 16, ...)$ is .....

(a) $2n$ (b) $4n$ (c) $2n$ (d) n^2

(29) The n^{th} term of the sequence $(-1, 4, -9, 16, ...)$ is .....

(a) $-n^2$ (b) $(-1)^n n^2$ (c) $(-n)^2$ (d) $-4n$

(30) The general term of the sequence $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...\right)$ is .....

(a) $\frac{n}{2n}$ (b) $\frac{n}{n+1}$ (c) $\frac{1}{n}$ (d) $\frac{n}{2n-1}$

(31) The general term of the sequence $\left(1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, ...\right)$ is T_n = ...

(a) n^{n-1} (b) $(10)^{-n}$ (c) $\left(\frac{1}{10}\right)^{n-1}$ (d) $(0.1)^{n+1}$

(32) The n^{th} term of the sequence $\left(2, 2, \frac{8}{3}, 4, ...\right)$ is .....

(a) $n - 1$ (b) $2n - 1$ (c) 2^{n-1} (d) $\frac{2^n}{n}$

(33) The general term of the sequence $\left(\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, ...\right)$ is .....

(a) $\frac{1}{n}$ (b) $\frac{1}{n-1}$ (c) $\frac{n}{n+1}$ (d) $\frac{n}{n+2}$

(34) The n^{th} term of the sequence $\left(\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, ...\right)$ is T_n = ...

(a) $\frac{1}{n^2+n}$ (b) $\frac{1}{n} + \frac{1}{n+1}$ (c) $\frac{1}{n^2+2}$ (d) $\frac{1}{n+3}$

(35) The general term of the sequence of the positive even integers is .....

(a) $4n$ (b) $2n$ (c) $2n-2$ (d) $4n-2$

(36) The general term of the sequence of the positive even integers greater than 2 is .....

(a) $2n$ (b) $2n+2$ (c) $4n$ (d) $4n-2$

(37) 📖 The general term of the sequence $(-1, 2, -4, 8, -16, ...)$ is .....

(a) $-(2)^{n-1}$ (b) $(-2)^{n-1}$

(c) $T_1 = -1, T_2 = 2, T_{n+2} = T_n T_{n+1}$ (d) $(-1)^n (2)^{n-1}$

(38) 📖 The general term of the sequence $((2\times3), (3\times4), (4\times5), (5\times6), ...)$ is T_n = ...

(a) $(n-1)(n+1)$ (b) $n(n+1)$ (c) $2n(n+1)$ (d) $(n+1)(n+2)$

(39) Which of the following can not be the general term of the sequence $(T_n) = (1, -1, 1, -1, ...)$?

(a) $T_n = -(-1)^n$ (b) $T_n = (-1)^{n-1}$ (c) $T_n = -\cos(n\pi)$ (d) $T_n = \cos(n\pi)$

(40) In the sequence $(2, 5, 8, 11, ...)$

First : The general term is T_n = ...

(a) $2n$ (b) $4n-2$ (c) $n+1$ (d) $3n-1$

Second : T_{11} = ...

(a) 22 (b) 32 (c) 34 (d) 28

Third : The order of the term whose value 59 is .....

(a) T_{15} (b) T_{22} (c) T_{18} (d) T_{20}

(41) In the sequence $(2, 4, 8, 16, ...)$

First : The general term is .....

(a) $2n$ (b) $n+1$ (c) $2(n+1)$ (d) 2^n

Second : The order of the term whose value 128 is .....

(a) T_5 (b) T_7 (c) T_8 (d) T_6

(42) The tenth term of the sequence $(\cos\pi, \cos\frac{\pi}{2}, \cos\frac{\pi}{3}, \cos\frac{\pi}{4}, ...)$ equals .....

(a) $\cos 10\pi$ (b) $\sin\frac{9\pi}{10}$ (c) $\cos\frac{2}{5}\pi$ (d) $\sin\frac{2\pi}{5}$

(43) 📖 In the sequence (T_n) where $T_n = 3n^2 - 1$, if $T_m = 74$, then m = ...

(a) ± 5 (b) 5 (c) 4 (d) 74

(44) In the sequence (T_n), if $T_n = a + bn$ and $T_1 = 5, T_3 = 11$, then $b - a$ = ...

(a) -1 (b) zero (c) 1 (d) 5

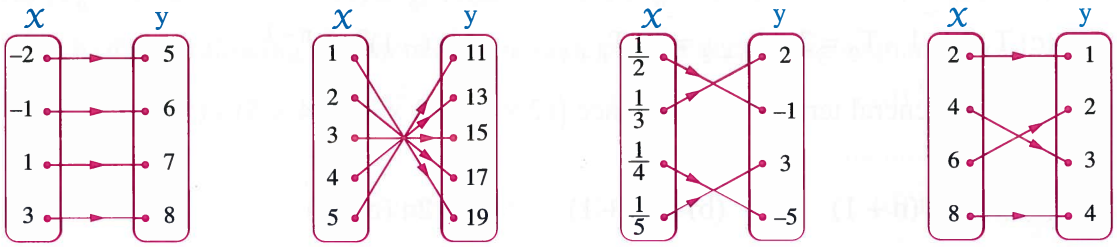
(45) The number of terms of the sequence $(1, 4, 9, 16, \ldots, 625)$ is .....

(a) 25 (b) 125 (c) 625 (d) 50

(46) In the sequence (T_n), if $T_1 = 1$, $T_{n+1} = 4T_n + k$, $T_3 = 31$, then $k =$ .....

(a) 2 (b) 5 (c) 3 (d) 4

(47) Which of the following function represents a real sequence ?



(a) (b) (c) (d)

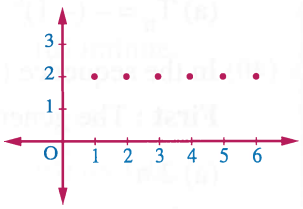
(48) The opposite figure represents ..... sequence.

(a) an increasing

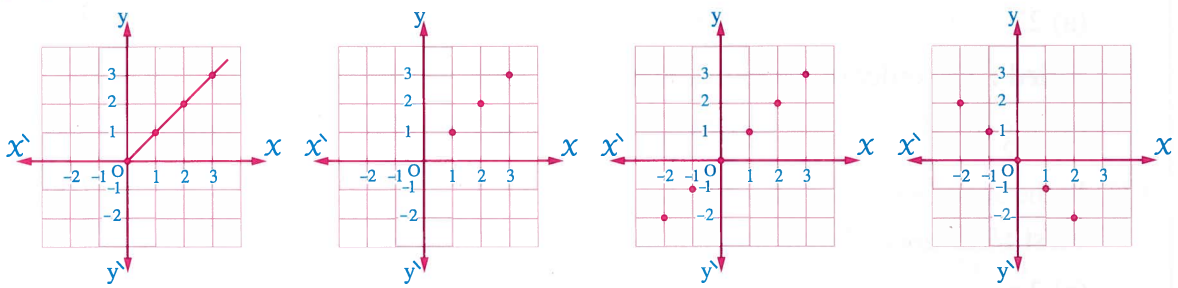
(b) a decreasing

(c) a constant

(d) an alternating



(49) Which of the following figures represent a sequence ?



(a) (b) (c) (d)

Second Essay questions

1 **Write the first five terms of each of the sequences whose general term is given by the following rules :**

(1) $T_n = 2n - 3$

(2) $T_n = \frac{(-1)^n}{n+3}$

(3) $T_n = \sin\left(\frac{n}{4}\pi\right)$

(4) $T_n = 1 + \frac{2}{\sqrt{n}}$

(5) $T_1 = 2$, $T_{n+1} = 2T_n$, $n \geq 1$

(6) $T_1 = -1$, $T_{n+1} = \frac{-1}{T_n}$, $n \geq 1$

2 **Show whether each of the sequences (T_n) is increasing , decreasing or otherwise :**

(1) $T_n = 2n + 5$

(2) 📖 $T_n = \frac{1}{3n - 1}$

(3) $T_n = \left(\frac{-1}{3}\right)^n$

(4) 📖 $T_n = \frac{(-1)^n}{2n} + 4$

3 **Write down the general term for each of the following sequences :**

(1) 📖 $(1 , 8 , 27 , 64 , ...)$

(2) 📖 $\left(\cos\frac{\pi}{3} , \cos\frac{2\pi}{3} , \cos\pi , \cos\frac{4\pi}{3} , ...\right)$

(3) $\left(\frac{1}{2} , \frac{3}{4} , \frac{9}{8} , \frac{27}{16} , ...\right)$

4 📖 In the sequence (T_n) , if $T_1 = 9$, $T_3 = 36$ and $T_{n+1} = T_n + n\,\chi$, find the value of χ

« 9 »

5 📖 **Discover the error :**

(1) Each function whose domain is $\mathbb{Z}$ is a sequence.

(2) Each function whose domain is $\{0 , 1 , 2 , 3 , 4 , ...\}$ is an infinite sequence.

(3) In the sequence (T_n) where $T_n = n^2$, then $T_n > T_{n+1}$

6 📖 **Activity :**

If (T_n) is a sequence whose terms are $(1 , 1 , 2 , 3 , 5 , 8 , 13 , ...)$

(1) Study the pattern of the sequence , then find the eighth and ninth terms.

(2) Represent the first nine terms of the sequence graphically.

(3) Do we consider such a sequence increasing , decreasing or otherwise ? Explain.

(4) Write down the relation between the terms of this sequence.

Life application

📖 **Sports : Kareem does physical fitness exercises for 8 minutes in the first day and increases the training period for two minutes daily.**

(1) Write the first five terms of this sequence.

(2) Find the general term of this sequence.

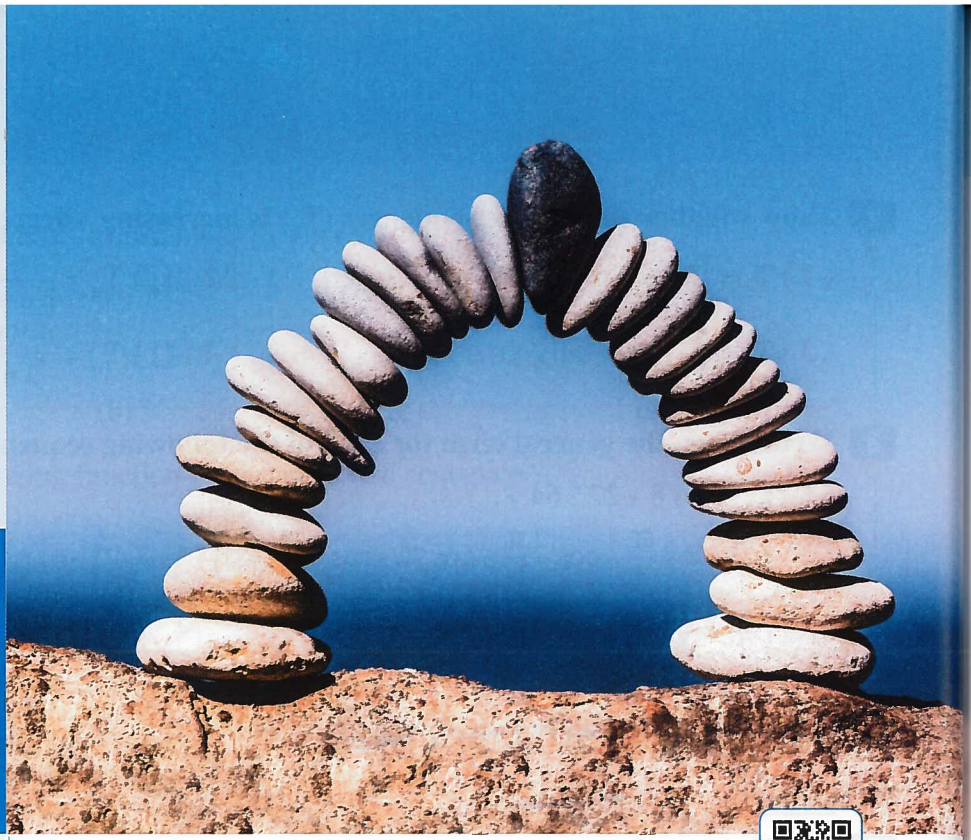
(3) How long does Kareem take in the seventh day ?

(4) In which day does Kareem take half an hour ? Explain.

Exercise

2

Series and summation notation



From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\sum_{r=1}^{5} 3 =$ .....

(a) 3 (b) 5 (c) 15 (d) 243

(2) $\sum_{r=2}^{5} 3 =$ .....

(a) 3 (b) 5 (c) 15 (d) 12

(3) $\sum_{r=1}^{4} r =$ .....

(a) 20 (b) 10 (c) 4 (d) 6

(4) $\sum_{r=1}^{10} r^2 =$ .....

(a) 10 (b) 55 (c) 220 (d) 385

(5) The value of the series $\sum_{r=7}^{23} 3r$ is .....

(a) 255 (b) 765 (c) 807 (d) 828

(6) The value of the series $\sum_{r=1}^{15} (r^2 + r + 1)$ is .....

(a) 1375 (b) 3720 (c) 14400 (d) 2232000

(7) The value of the series $\sum_{n=1}^{10} (-1)^n$ = ...

(a) zero (b) 1 (c) – 1 (d) 10

(8) If $\sum_{r=1}^{n} (r) = 136$, then $\sum_{r=1}^{n} (4r)$ = ...

(a) 140 (b) 544 (c) 272 (d) 34

(9) $\sum_{r=1}^{10} (3r+2) + \sum_{r=11}^{35} (3r+2)$ = ...

(a) $\sum_{r=1}^{35} (3r+2)$ (b) $\sum_{r=1}^{35} (3r+2)^2$ (c) $\sum_{r=1}^{35} (6r+4)$ (d) $\sum_{r=1}^{35} (3r+4)$

(10) If $\sum_{r=1}^{n} (100 - 4r) = \text{zero}$, then n = ...

(a) 25 (b) 49 (c) 50 (d) 98

(11) If $\sum_{r=1}^{n} (r^2) = 55$, $\sum_{r=1}^{n} (2r-1) = 25$, then $\sum_{r=1}^{n} (r^2 + 2r - 1)$ = ...

(a) 55 (b) 30 (c) 80 (d) 1375

(12) If $\sum_{r=1}^{25} (3 + kr) = 100$, then k = ...

(a) 25 (b) 13 (c) $\frac{1}{13}$ (d) $\frac{1}{25}$

(13) The sum of the positive multiples of the number 6 and less than 100 is .....

(a) $\sum_{r=1}^{100} 6r$ (b) $\sum_{r=1}^{16} 6r$ (c) $\sum_{r=1}^{16} r$ (d) $\sum_{r=1}^{6} (100 - 6r)$

(14) The twentieth term in the series $(2 \times 4 + 4 \times 6 + 6 \times 8 + ...)$ equals .....

(a) 1680 (b) 1600 (c) 840 (d) 420

(15) $1 + 2 + 3 + 4 + 5 + ... + 20$ by using summation notation = ...

(a) $\sum_{r=1}^{20} 1$ (b) $\sum_{r=1}^{20} r^2$ (c) $\sum_{r=1}^{20} (r+1)$ (d) $\sum_{r=1}^{20} r$

(16) $2 + 4 + 6 + 8 + ... + 30$ = ...

(a) $\sum_{r=1}^{30} r$ (b) $\sum_{r=1}^{15} r$ (c) $\sum_{r=1}^{15} 2r$ (d) $\sum_{r=1}^{30} 2r$

(17) $1^2 + 2^2 + 3^2 + 4^2 + ... + 8^2$ = ...

(a) $\sum_{r=1}^{8} r^2$ (b) $\sum_{r=1}^{64} r$ (c) $\sum_{r=1}^{8} 2r$ (d) $\sum_{r=1}^{8} 2r^2$

(18) $3 + 6 + 9 + 12 + ...$ = ...

(a) $\sum_{r=1}^{\infty} 3r$ (b) $\sum_{r=1}^{\infty} (r+3)$ (c) $\sum_{r=1}^{\infty} (r+2)$ (d) $\sum_{r=1}^{\infty} r^3$

(19) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ by using summation notation is .....

(a) $\sum_{r=1}^{\infty} \left(\frac{1}{2}\right)^r$ (b) $\sum_{r=1}^{5} \left(\frac{1}{2}\right)^{r-1}$ (c) $\sum_{r=1}^{\infty} \left(\frac{1}{2}\right)^{r-2}$ (d) $\sum_{r=1}^{5} \left(\frac{1}{2}\right)^{r-2}$

(20) $-1 - 8 - 27 - 64 \ldots - 216$ by using summation notation is .....

(a) $\sum_{r=1}^{216} (-r)^3$ (b) $\sum_{r=1}^{15} -r^3$ (c) $\sum_{r=1}^{6} (-1)^r (r)^3$ (d) $\sum_{r=1}^{6} (-r)^3$

(21) $(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + \ldots$ equals .....

(a) $\sum_{r=1}^{\infty} r(r+1)$ (b) $\sum_{r=1}^{\infty} r(r-1)$

(c) $\sum_{r=1}^{\infty} (r+1)(r+2)$ (d) $\sum_{r=1}^{\infty} (r-1)(r+1)$

(22) The series $(2 \times 3) + (3 \times 4) + (4 \times 5) + \ldots$ by using summation notation = ...

(a) $\sum_{r=1}^{\infty} (r+1)(r+2)$ (b) $\sum_{r=2}^{\infty} r(r+1)$

(c) $\sum_{r=3}^{\infty} r(r-1)$ (d) All the previous.

(23) The series $(7 \times 1) + (7 \times 2) + (7 \times 3) + \ldots + (7 \times 20)$ by using summation notation = ...

(a) $7 \sum_{r=1}^{140} r$ (b) $\sum_{r=1}^{n} 7r$ (c) $\sum_{r=1}^{20} 7 \cdot \sum_{r=1}^{20} r$ (d) $\sum_{r=1}^{20} 7r$

(24) $(1 \times 2 \times 3) + (3 \times 4 \times 5) + (5 \times 6 \times 7) + \ldots$ by using summation notation equals .....

(a) $\sum_{r=1}^{\infty} r(r+1)(r+2)$ (b) $\sum_{r=1}^{7} r(r+1)(r+2)$

(c) $\sum_{r=1}^{5} (2r-1)(2r)(2r+1)$ (d) $\sum_{r=1}^{\infty} (2r-1)(2r)(2r+1)$

(25) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \ldots + \frac{1}{20}$ by using summation notation = ...

(a) $\sum_{r=1}^{20} \frac{1}{2r}$ (b) $\sum_{r=1}^{\infty} \frac{1}{r+1}$ (c) $\sum_{r=1}^{10} \frac{1}{2r}$ (d) $\sum_{r=0}^{20} \frac{1}{r+2}$

(26) $-\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \ldots$ by using summation notation = ...

(a) $\sum_{r=1}^{\infty} -1 \cdot \sum_{r=1}^{\infty} 2^{-r}$ (b) $\sum_{r=0}^{16} (-1)^r \left(\frac{1}{2}\right)^{r+1}$

(c) $\sum_{r=1}^{\infty} 2^{-r}$ (d) $\sum_{r=1}^{\infty} \left(-\frac{1}{2}\right)^r$

(27) By using summation notation you can express the series $9 + 99 + 999 + 9999 + \ldots$ to the n^{th} term by .....

(a) $\sum_{r=1}^{\infty} (10^r \times 9)$ (b) $\sum_{r=1}^{n} (9 + 10r)$

(c) $\sum_{r=0}^{n} (9 + 10r)$ (d) $\sum_{r=1}^{n} (10^r - 1)$

(28) If $a = \sum_{r=1}^{100} r$, $b = \sum_{r=2}^{101} r$, $c = \sum_{r=1}^{99} r$, then .....

(a) $a < b < c$ (b) $a < c < b$ (c) $c < a < b$ (d) $c < b < a$

(29) If $C_1 = 1 + 2 + 3 + \ldots + n$, $C_2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$, then .....

(a) $C_1 = C_2^2$ (b) $C_2 = C_1^2$ (c) $\frac{C_2}{C_1} = \frac{2n+1}{3}$ (d) $\frac{C_2}{C_1} = n$

(30) If $\sum_{r=1}^{n} r = 55$, then $\sum_{r=1}^{n} r^2 =$ .....

(a) 385 (b) 506 (c) 1115 (d) 3025

(31) $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots + n^2 + n =$ .....

(a) $\frac{n(n+1)}{2}$ (b) $\left(\frac{n(n+1)}{2}\right)^2$

(c) $\frac{n(n+1)(n+2)}{3}$ (d) $\frac{n(n+1)(n+2)(n+3)}{4}$

(32) The sum of areas of squares with side lengths 3 , 4 , 5 , ... , 10 cm. = ... cm^2

(a) 360 (b) 380 (c) 340 (d) 350

(33) If the sum of the first n terms of the positive integers equals $\frac{1}{5}$ the sum of the squares of these terms , then n = ...

(a) 5 (b) 6 (c) 7 (d) 8

(34) The sum of the first 2022 terms of the sequence (1 , 2 , 3 , 4 , 1 , 2 , 3 , 4 , ...) equals .....

(a) 5051 (b) 5052 (c) 5053 (d) 5054

(35) Sum of 1^{st} seven terms from the sequence (3 , 7 , 11 , 15 , ...) equals .....

(a) $\sum_{r=1}^{7} (r + 1)$ (b) $\sum_{r=1}^{7} (2r + 1)$ (c) $\sum_{r=1}^{7} 3(r - 1)$ (d) $\sum_{r=1}^{7} (4r - 1)$

Second Essay questions

1 **Expand each of the following series :**

(1) $\sum_{r=1}^{5} (3r - 2)$

(2) $\sum_{r=1}^{8} \left((-1)^r + 4r\right)$

(3) $\sum_{r=1}^{\infty} \left(\left(\frac{1}{2}\right)^r - 1\right)$

(4) $\sum_{r=1}^{\infty} \left(\frac{1}{r} - \frac{1}{r+1}\right)$

2 **Expand each of the following series , then find the expansion sum :**

(1) $\sum_{r=1}^{4} (r^2)$

(2) $\sum_{r=1}^{5} (1 - r^2)$

(3) $\sum_{r=1}^{5} \left(\frac{-1}{2}\right)^r$

(4) $\sum_{r=1}^{n} \left(\frac{1}{r+1} - \frac{1}{r}\right)$

3 **Find in two different methods :**

(1) $\sum_{r=1}^{5} (2r^2 - 3r + 5)$

(2) $\sum_{r=1}^{4} (3 - 2r + r^2)$

4 **If you know that** $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ **and** $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ **, use the properties of the summation notation Σ to find the value of each of the following :**

(1) $\sum_{r=1}^{n} (r + 2)$

(2) $\sum_{r=1}^{6} (2r^2 - 1)$

(3) $\sum_{r=8}^{12} 2(r + 5)$

(4) $\sum_{r=4}^{7} (r^2 - 2r + 3)$

5 **Discover the error :**

(1) The series is a function whose domain is the set of the positive integers or a subset of it.

(2) $\sum_{r=0}^{5} (2r + 1) = 1 + 3 + 5 + 7 + 9$

Third Higher skills

Choose the correct answer from those given :

(1) If $\sum_{r=1}^{20} \sin\theta = 10$, then one of the values of θ is .....

(a) 30° (b) 60° (c) 90° (d) 120°

(2) If χ_1 , χ_2 are the roots of the equation : $\chi^2 - (m-2)\chi - 1 = 0$ and $\sum_{r=1}^{2} \chi_r = 3$, then m = ...

(a) 2 (b) 3 (c) 4 (d) 5

(3) If $\sum_{r=1}^{n} (3) = 12$ and $f(\chi) = 3 - 2\chi + \chi^2$, then $\sum_{r=1}^{n} f(r)$ = ...

(a) 12 (b) 22 (c) 52 (d) 62

(4) If $\sum_{x=1}^{m} (\chi) + \sum_{y=1}^{m} (y) + \sum_{z=1}^{m} (z) = 165$, then m = ...

(a) 7 (b) 8 (c) 9 (d) 10

(5) If $\sum_{n=1}^{k} (n+1)^2 = 84 + \sum_{n=1}^{k} (n-1)^2$, then k = ...

(a) 4 (b) 6 (c) 7 (d) 9

(6) If $\sum_{k=1}^{n} k = \mathcal{X}$, $\sum_{k=1}^{n} k^2 = 7\mathcal{X}$, then n = ...

(a) 8 (b) 10 (c) 12 (d) 15

(7) $\sum_{\mathcal{X}=1}^{90} [\cos(\mathcal{X}°) - \sin(\mathcal{X}°)]$ = ...

(a) – 1 (b) 1 (c) 0 (d) 90

(8) If $\sum_{\mathcal{X}=1}^{6} (a\mathcal{X} + b) = 15$, $\sum_{\mathcal{X}=2}^{6} (a\mathcal{X} + b) = 7$, then a + b = ...

(a) – 5 (b) – 4 (c) 4 (d) 8

(9) If $\sum_{r=1}^{n} \log\left(\frac{r+1}{r}\right) = 2$, then n = ...

(a) 10 (b) 20 (c) 99 (d) 100

(10) If $\mathcal{X} = \sum_{r=1}^{n} 3r$, $y = \sum_{r=1}^{n} (3r + 2)$ and $y - \mathcal{X} = 10$, then n = ...

(a) 5 (b) 8 (c) 11 (d) 13

(11) If n > 8 and $\sum_{r=5}^{n} r^2 = \mathcal{X}$, $\sum_{r=5}^{8} r^2 + \sum_{r=8}^{n} r^2 = y$, then .....

(a) $\mathcal{X} = y$ (b) $\mathcal{X} = y + 64$ (c) $y = \mathcal{X} + 64$ (d) $y = 64\mathcal{X}$

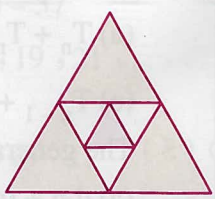
(12) $\sum_{a=1}^{2} \sum_{b=1}^{2} \sum_{c=1}^{2} abc$ = ...

(a) 8 (b) 27 (c) 64 (d) 125

Life applications

1 A child wants to build a pyramid of identical wooden cubes , where its top consists of one cube , the second row consists of two cubes and the third row consists of three cubes and so on. Express the number of cubes used in building the pyramid using the summation notation Σ , known that the pyramid consists of 10 rows , and find this number. « 55 cubes »

2 **Geometry : The figure opposite represents an equilateral triangle whose side length is 32 cm. Its sides are bisected and the interior triangle is drawn and this pattern lasts interiorly till we get three triangles including the first triangle.**



(1) Write down the series of the three triangle perimeters using the summation notation.

(2) Find the sum of the three triangle perimeters in centimetres. « 168 cm. »

Exercise

3

Arithmetic sequence

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

Identifying arithmetic sequence

(1) All the following sequences can be arithmetic except .....

(a) $(3 , 7 , 11 , 15 , ...)$ (b) $(-11 , -15 , -19 , -23 , ...)$

(c) $\left(\frac{1}{2} , \frac{1}{4} , \frac{1}{5} , \frac{1}{6} , ...\right)$ (d) $\left(\frac{21}{5} , \frac{16}{5} , \frac{11}{5} , \frac{6}{5} , ...\right)$

(2) All the following sequences are arithmetic except .....

(a) $(2n - 5)$ (b) $(43 - 7n)$ (c) (2^n) (d) $(3n)$

(3) From the following sequences , the arithmetic sequence is .....

(a) $(T_n) = \left(\frac{n+1}{n}\right)$ (b) $(T_n) = \left((n+1)^2\right)$

(c) $(T_n) = \left(\frac{3}{n}(n+2)\right)$ (d) $(T_n) = \left(\frac{n^3 - 1}{n^2 + n + 1}\right)$

(4) The sequence (T_n) is an arithmetic sequence if and only if ..... for $n > 1$

(a) $T_n + T_{n+1} =$ constant. (b) $\frac{T_{n+1}}{T_n} =$ constant.

(c) $T_{n-1} + T_{n+1} = 2T_n$ (d) $T_{n+1} - T_n = \frac{1}{n}$

(5) The general term of the sequence $(a , a + d , a + 2d , ...)$ is .....

(a) $dn + (d - a)$ (b) $a + nd$ (c) $a + (n + 1)d$ (d) $dn + (a - d)$

(6) The n^{th} term of the arithmetic sequence $(3 , 5 , 7 , ...)$ equals .....

(a) $3n$ (b) $2n + 1$ (c) $5n - 2$ (d) 2^n

(7) If $2a+1$, $5a-1$ and $6a+3$ are three consecutive terms of an arithmetic sequence , then a = ...

(a) 1 (b) 2 (c) 3 (d) 5

(8) If $(29 , x , ... , 3x , 95)$ is an arithmetic sequence , then x = ...

(a) 30 (b) 31 (c) 32 (d) 33

(9) If 36 , a , 24 and b are consecutive terms of an arithmetic sequence , then b = ...

(a) 12 (b) 16 (c) 18 (d) 24

(10) If $(x , y , 3x+y , x+3y+2 , ...)$ is an arithmetic sequence , then $x-y$ = ...

(a) -6 (b) -2 (c) 2 (d) 6

(11) The number of terms of the sequence $(2 , 8 , 14 , ... , 68)$ equals ..... terms.

(a) 6 (b) 8 (c) 12 (d) 16

(12) The last term in the arithmetic sequence whose first term = 3 and its common difference = 5 and has 25 terms equals .....

(a) 113 (b) 118 (c) 123 (d) 128

(13) The tenth term in the arithmetic sequence $(\sqrt{3} , \sqrt{12} , \sqrt{27} , ...)$ equals .....

(a) $\sqrt{243}$ (b) $\sqrt{300}$ (c) $\sqrt{363}$ (d) $\sqrt{432}$

(14) The value of the middle term of the sequence $(2 , 5 , 8 , 11 , ... , 128)$ is .....

(a) 22 (b) 43 (c) 65 (d) 96

(15) In the arithmetic sequence $(12 , 14 , 16 , ...)$, the order of the term whose value 102 is .....

(a) T_{26} (b) T_{48} (c) T_{46} (d) T_{45}

(16) In the arithmetic sequence $(22 , 20 , 18 , ...)$, the order of the term whose value equals zero is .....

(a) 8 (b) 10 (c) 12 (d) 14

(17) The value of T_{15} from the end in the arithmetic sequence $(19 , 15 , 11 , ... , -61)$ equals .....

(a) -8 (b) -5 (c) -1 (d) -37

(18) The value of the tenth term from the end in the arithmetic sequence $(17 , 19 , 21 , ... , 47)$ equals .....

(a) 29 (b) 31 (c) 25 (d) 27

(19) The number of terms in an arithmetic sequence is 20 , then the fourth term from the end is the ..... term from the beginning.

(a) 15 (b) 16 (c) 17 (d) 18

(20) The order of the first term smaller than – 180 in the arithmetic sequence (64 , 61 , 58 , ...) is .....

(a) 81 (b) 82 (c) 83 (d) 84

(21) The value of the first term greater than 1000 from the arithmetic sequence (2 , 9 , 16 , ...) is .....

(a) 1003 (b) 1004 (c) 1005 (d) 1006

(22) The order of the first positive term in the arithmetic sequence (– 48 , – 45 , – 42 , ...) is .....

(a) T_{16} (b) T_{17} (c) T_{18} (d) T_{19}

(23) The order of the last negative term in the arithmetic sequence (– 62 , – 57 , – 52 , ...) is .....

(a) T_{12} (b) T_{13} (c) T_{14} (d) T_{11}

(24) The order of the last positive term in the sequence (28 , 25 , 22 , ...) is .....

(a) T_9 (b) T_{10} (c) T_{11} (d) T_{12}

(25) The value of the first negative term from the arithmetic sequence (341 , 334 , 327 , ...) equals .....

(a) – 4 (b) – 3 (c) – 2 (d) – 1

(26) If $(T_n) = (2 , 5 , 8 , ... , \ell)$ is an arithmetic sequence whose seventeenth term from the beginning is itself the seventeenth term from the end , then ℓ = ...

(a) 98 (b) 96 (c) 101 (d) 95

(27) If (T_n) is an arithmetic sequence , $T_3 - T_5 = 6$, $T_4 = 16$, then the value of the first negative term in the sequence equals .....

(a) – 4 (b) – 3 (c) – 2 (d) – 1

(28) The order of the first term more than 100 in the sequence $(T_n) = (2 + 5\ n)$ is .....

(a) 19 (b) 20 (c) 21 (d) 22

(29) The first term in an arithmetic sequence = 4 and its fifth term = 20 , then the sequence is .....

(a) (4 , 8 , 12 , ...) (b) (4 , 6 , 8 , ...) (c) (4 , 20 , 36 , ...) (d) (4 , 24 , 44 , ...)

(30) An arithmetic sequence whose first term is 5 , $T_{n+1} = T_n + 3$, then its fifth term equals .....

(a) 12 (b) 20 (c) 17 (d) 19

(31) If the third term is an arithmetic sequence = 12 and its seventh term = 24 , then the tenth term = ...

(a) 36 (b) 39 (c) 30 (d) 33

(32) If T_2 from an arithmetic sequence is 129 , T_5 from the same sequence is 141 , then the order of the term whose value 161 is .....

(a) 3 (b) 9 (c) 10 (d) 12

(33) The sixth term from an arithmetic sequence = 34 , the sum of the seventh and ninth terms equals 88 , then the order of first term greater than 105 in this sequence is .....

(a) 14 (b) 16 (c) 18 (d) 21

(34) If (T_n) is an arithmetic sequence , $T_{13} + T_{18} = 68$, then $T_{14} + T_{17} =$ .....

(a) 68 (b) 136 (c) 34 (d) 204

(35) In the arithmetic sequence $(2a + 5b , 3a + 8b , 4a + 11b , ...)$, then $T_{10} =$ .....

(a) $a + 9b$ (b) $10a + 30b$ (c) $11a + 32b$ (d) $12a + 35b$

(36) If (T_n) is an arithmetic sequence in which $\frac{T_4}{T_7} = \frac{2}{3}$, then $\frac{T_6}{T_8} =$ .....

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\frac{6}{7}$ (d) $\frac{3}{4}$

(37) In any arithmetic sequence (T_n) , $\frac{T_{45} + T_{51}}{T_{48}} =$ .....

(a) 2 (b) 3 (c) 4 (d) 5

(38) An arithmetic sequence in which $T_n = m$, $T_m = n$, where $m \neq n$, then the common difference of the sequence (d) = ...

(a) 1 (b) 2 (c) – 1 (d) – 2

(39) In an arithmetic sequence , if $T_{17} = 73$, $T_{73} = 17$, then the order of the term whose value = 0 , is .....

(a) 56 (b) 89 (c) 90 (d) 91

(40) If (T_n) is an arithmetic sequence with common difference (d) , then d = ...

(a) $T_m - T_r$ (b) $\frac{T_m - T_r}{m - r}$ (c) $\frac{T_m}{T_r}$ (d) T_{m-r}

(41) If T_{3n-2} in the sequence $\left(13 , 14\frac{1}{4} , 15\frac{1}{2} , ...\right)$ equals T_{2n+1} in the sequence $\left(19 , 20\frac{1}{2} , 22 , ...\right)$, then n = ...

(a) 15 (b) 14 (c) 13 (d) 12

(42) The number of 3-digit numbers which are divisible by 9 is .....

(a) 90 (b) 98 (c) 99 (d) 100

(43) If the measures of the angles of a pentagon form an arithmetic sequence whose common difference = 10° , then the measure of the greatest angle of this pentagon equals .....

(a) 88° (b) 108° (c) 118° (d) 128°

(44) If a , b , c form an arithmetic sequence , then $\frac{(a - c)^2}{(b^2 - ac)} =$ .....

(a) – 1 (b) 1 (c) 2 (d) 4

(45) The tenth common term of the two sequences $(3 , 7 , 11 , ...) , (1 , 6 , 11 , ...)$ equals .....

(a) 191 (b) 193 (c) 211 (d) 231

(46) Arithmetic sequence in which $T_n = m^2$, $T_m = n^2$ where $m \neq n$, then the common difference of the sequence (d) = ...

(a) $m^2 + n^2 - 2$ (b) $m + n$ (c) $-m - n$ (d) $-m + n$

(47) If (T_n) is an arithmetic sequence in which $m\,T_m = nT_n$, $m \neq n$ then T_{m+n} = ...

(a) zero (b) $m + n - 1$ (c) 1 (d) $m - n$

(48) If the lengths of the sides in a right-angled triangle form an arithmetic sequence , then the common difference = ... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(49) If the first term in an arithmetic sequence is equal to 2 and the quotient of division of its seventh term by its third term is equal to 2 and remainder is 8 , then its fourteenth term is equal to .....

(a) 76 (b) 67 (c) 28 (d) 54

The arithmetic means

(50) The arithmetic mean of the two numbers 8 and 12 is .....

(a) 8 (b) 12 (c) 20 (d) 10

(51) If the arithmetic mean of the two numbers x and 26 is 21 , then x = ...

(a) 26 (b) 16 (c) 42 (d) 21

(52) If $x < 0$, and $(7x , 8 , x^2 - 2)$ form an arithmetic sequence , then x = ...

(a) 2 (b) -9 (c) -2 (d) -7

(53) Arithmetic mean of the two numbers $a + b$ and $a - b$ equals .....

(a) a (b) b (c) 2 a (d) 2 b

(54) In an arithmetic sequence (T_n) : $\frac{T_{n-1} + T_{n+1}}{T_n}$ = ...

(a) T_{n+2} (b) T_{2n} (c) zero (d) 2

(55) In an arithmetic sequence , the fifth mean is the ..... term.

(a) fifth (b) fourth (c) tenth (d) sixth

(56) If (a , 6 , b , ...) form an arithmetic sequence , then .....

(a) $a + b < 6$ (b) $6 < a + b < 12$ (c) $a + b = 6$ (d) $a + b = 12$

(57) The first term in an arithmetic sequence is (a) and its common difference is (d) , then each of the following is true except .....

(a) $T_3 + T_5 = 2\,T_4$ (b) $T_{10} + T_{12} = 2\,T_{11}$

(c) $T_3 + T_7 + T_{17} = 3\,T_9$ (d) $T_4 + T_6 = T_{10}$

(58) If a and b are two arithmetic means between x and y , then $\frac{y - x}{b - a}$ = ...

(a) 2 (b) 3 (c) 4 (d) 6

(59) If (a^2 , b^2 , c^2) form an arithmetic sequence , then $b^2 =$ .....

(a) $a^2 + c^2$ (b) $2(a^2 + c^2)$ (c) $\frac{a^2 + c^2}{2}$ (d) $\sqrt{a^2 + c^2}$

(60) The arithmetic mean between the two numbers $(a + b)^2 , (a - b)^2$ is .....

(a) $2a$ (b) $a + b$ (c) $a^2 + b^2$ (d) $a^2 - b^2$

(61) The sum of the first and the last arithmetic means between the two numbers 7 , 31 equals .....

(a) 19 (b) 38 (c) 24 (d) 13

(62) If $(a , b , c , 20 , x , y , z)$ form an arithmetic sequence , then $a + b + c + x + y + z =$ .....

(a) 60 (b) 80 (c) 100 (d) 120

(63) If you insert n arithmetic means between a , b , then the common difference of the arithmetic sequence is .....

(a) $\frac{a - b}{n + 1}$ (b) $\frac{a - b}{n + 2}$ (c) $\frac{b - a}{n + 1}$ (d) $\frac{b - a}{n + 2}$

(64) When you insert a number of arithmetic means between a and ℓ , then the last mean is ..... (where "d" is the common difference of resulting sequence)

(a) $\ell - d$ (b) ℓ (c) $\ell - 2d$ (d) $\ell + d$

(65) When you insert 10 arithmetic means between 5 and 38 , then the resulted sequence is .....

(a) (5 , 10 , 15 , ... , 38) (b) (5 , 7 , 9 , 11 , ... , 38)

(c) (5 , 8 , 11 , ... , 38) (d) (5 , 9 , 13 , ... , 38)

(66) If you insert 12 arithmetic sequence between – 14 and 51 , then the seventh mean = ...

(a) 21 (b) 16 (c) 26 (d) 31

(67) If you insert number of arithmetic means between – 65 and – 125 and the twelveths mean = – 113 , then the number of means = ...

(a) 12 (b) 13 (c) 14 (d) 15

(68) Several arithmetic means are inserted between 8 and 62. If the sum of the 2^{nd} and 6^{th} means = 40 , then the number of means = ...

(a) 15 (b) 16 (c) 17 (d) 18

(69) The difference between two numbers is 5 and their arithmetic mean is 6.5 , then the two numbers are .....

(a) 5 , 10 (b) – 1 , 4 (c) 2 , 7 (d) 9 , 4

(70) Two numbers , one of them is 2 more than the twice of the other. If their arithmetic mean is $14\frac{1}{2}$, then the two numbers are .....

(a) 6 , 14 (b) 11 , 18 (c) 9 , 20 (d) 10 , 22

(71) Two numbers , the ratio between them = 3 : 10 and their arithmetic mean = 13 , then the two numbers are .....

(a) 3 , 10 (b) 9 , 15 (c) 9 , 30 (d) 6 , 20

(72) If the arithmetic mean between a and b is 8 and the arithmetic mean between 4 a and 2 b is 20 , then (a , b) = ...

(a) (4 , 12) (b) (6 , 10) (c) (5 , 11) (d) (8 , 8)

(73) The arithmetic mean of two numbers is 23 , their product is 493 , then these two numbers are .....

(a) 15 , 31 (b) 20 , 26 (c) 16 , 30 (d) 29 , 17

(74) If a and b are two arithmetic means between x and y , then $\frac{y^2 - x^2}{b^2 - a^2}$ = ...

(a) 1 (b) 2 (c) 3 (d) 4

(75) If (T_n) is an arithmetic sequence where $T_n = 3n - 2$, then the arithmetic mean between T_5 and T_{11} equals .....

(a) 8 (b) 16 (c) 22 (d) 26

(76) If $(1 , x , y)$ form an arithmetic sequence and $y^2 = x , x \neq y \neq 1$, then y = ...

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(77) If $(x , 4x + 1 , x + 2y , 3y + 2)$ form an arithmetic sequence , then x = ...

(a) 2 (b) – 2 (c) – 7 (d) 7

(78) If (a , b , c , d , ...) is an arithmetic sequence and b = 8 , then the arithmetic mean of the numbers a , b , c equals .....

(a) 6 (b) 8 (c) 16 (d) 24

(79) (T_n) is an arithmetic sequence in which $T_5 = x$, then $\sum_{r=2}^{8} T_r$ = ...

(a) x (b) $7x$ (c) $\frac{x}{7}$ (d) $x + 7$

Second Essay questions

Exercise on the arithmetic sequence

1 Show which of the following sequences is arithmetic and which is not , then find the general term of the arithmetic sequence :

(1) (9 , 13 , 17 , 21 , ...)

(2) (4 , 7 , 12 , 19 , ...)

(3) (– 12 , – 18 , – 24 , – 30 , – 36)

(4) $\left(\frac{1}{2} , \frac{1}{3} , \frac{1}{4} , \frac{1}{5} , ...\right)$

(5) $(x , x + y , x + 2y , ...)$

(6) (7 , 7 , 7 , 7 , 7)

2 **Show which of the following sequences is arithmetic , find its common difference and write down its first three terms :**

(1) $(T_n) = (5n + 2)$

(2) $(T_n) = \left(\frac{4 - 5n}{2}\right)$

(3) $(T_n) = (3 \times 7^{n+1})$

(4) $(T_n) = \left(\frac{n^2 - 25}{n + 5}\right)$

3 **In the arithmetic sequence (63 , 59 , 55 , ... , – 133) , find :**

(1) The value of the seventh term. (2) The number of the sequence terms. « 39 , 50 »

4 Find the order and the value of the first negative term in the arithmetic sequence (35 , 31 , 27 , ...) « 10 , – 1 »

5 Find the order and the value of the first positive term in the arithmetic sequence (– 51 , – 48 , – 45 , ...) « 19 , 3 »

6 Find the order and the value of the last negative term in the arithmetic sequence (– 39 , – 34 , – 29 , ...) « 8 , – 4 »

7 Find the number of negative terms in the arithmetic sequence (– 47 , – 42 , – 37 , ...) « 10 »

8 Find the number of positive terms in the arithmetic sequence (72 , 63 , 54 , ...) « 8 »

9 **Prove that :** 100 does not belong to the arithmetic sequence (13 , 17 , 21 , ...)

10 If (log x , log y , log z , ...) is an arithmetic sequence , then prove that $y^2 = x z$ (where x , y , $z \in \mathbb{R}^+$)

11 Let (12 , x , ... , y , – 24) be an arithmetic sequence such that its last term equals three times its sixth term , then find the value of each of x and y and the number of terms of the arithmetic sequence. « 8 , – 20 , 10 »

12 **Discover the error :**

(1) The common difference of the arithmetic sequence is defined as the difference between a term and the term directly previous to it ***i.e.*** $d = T_n - T_{n-1}$ for each $n \in \mathbb{Z}^+$

(2) The relation between n and T_n of an arithmetic sequence is given as follows : $T_n = a n + b$ where a and b are two constants and b is the common difference of the sequence in this relation.

Exercises on identifying the arithmetic sequence

13 Find the arithmetic sequence in which $T_8 = 11$ and T_{10} is the additive inverse of T_{17} « (25 , 23 , 21 , ...) »

14 Find the arithmetic sequence whose fourth term = 11 and the sum of the fifth and ninth terms = 40 , then find the order of the term whose value is 152 in this sequence.
« (2 , 5 , 8 , ...) , 51 »

15 Find the arithmetic sequence in which $T_2 + T_4 = 4$ and $T_6 + T_7 + T_8 = 54$ « (– 6 , – 2 , 2 , ...) »

16 Find the arithmetic sequence in which $T_5 = 2$ and $T_7 = 8$ and find the order and the value of the first term which exceeds 143
« (– 10 , – 7 , – 4 , ...) , T_{53} , 146 »

17 Find the arithmetic sequence in which $T_1 - T_6 = 25$ and $T_7 + T_8 = 95$ and find the order and the value of the first negative term in this sequence.
« (80 , 75 , 70 , ...) , T_{18} , – 5 »

18 Find the arithmetic sequence whose sum of its fifth and tenth terms equals 22 and its eighth term is 3 times of its fourth term.
« (– 2 , 0 , 2 , 4 , ...) »

19 Find the arithmetic sequence whose sixth term = 20 and the ratio between the fourth and tenth terms is 4 : 7
« (10 , 12 , 14 , ...) »

20 Find the arithmetic sequence in which the sum of its T_2 and T_5 equals 4 and the product of its T_3 and T_6 equals 7 , show that there are two arithmetic sequences.
« $\left(-1 , \frac{1}{5} , \frac{7}{5} , ...\right)$, (– 3 , – 1 , 1 , ...) »

21 (T_n) is an arithmetic sequence in which $T_2 + T_4 = 42$ and $T_3 \times T_5 = 315$, find this sequence.
« (27 , 24 , 21 , ...) »

22 An increasing arithmetic sequence in which the sum of its second and its third terms equals 15 and $T_5^2 = 225$, find the arithmetic sequence.
« (3 , 6 , 9 , ...) »

23 Find the arithmetic sequence in which its terms are all positive and the product of its first and its fourth terms equals 45 and $T_3 T_{10} - T_4 T_7 = 24$
« (3 , 7 , 11 , ...) »

24 Find the arithmetic sequence such that the number of its terms is 21 and its middle term equals 32 and the sum of its last three terms equals 177
« (2 , 5 , 8 , ...) »

25 Find the arithmetic sequence in which the sum of its second and third terms equals – 7 and the sum of their squares equals 29
« (– 8 , – 5 , – 2 , ...) or (1 , – 2 , – 5 , ...) »

26 Four numbers form an arithmetic sequence , their sum equals 36 and the sum of their squares equals 344 , find these numbers.
« 6 , 8 , 10 , 12 »

27 Three numbers form an arithmetic sequence , their sum is 33 and their product is 792 , find these numbers.
« 4 , 11 , 18 »

28 Find the number of the integers lying between 110 and 450 such that each of them is divisible by 11 « 30 »

29 What is the value of n in the arithmetic sequence whose first term = 3 , $T_n = 39$ and $T_{2n} = 79$? Find the sequence. « 10 , (3 , 7 , 11 , ...) »

30 A finite arithmetic sequence in which $T_1 = 7$ and T_{11} from the beginning is 47 and T_{11} from the end is 395 , find (T_n) « (7 , 11 , 15 , ... , 435) »

31 Find the arithmetic sequence in which $T_1 = 8$ and $T_{3n} = 3\,T_n$ « (8 , 16 , 24 , ...) »

32 Four numbers in an arithmetic sequence whose sum is 44 and if we add 3 to the second number , then the first , second and fourth numbers form other arithmetic sequence , find the four numbers. « 2 , 8 , 14 , 20 »

33 If (x , y , z) form an arithmetic sequence **prove that :** $(3x + 1 , 3y + 1 , 3z + 1)$ is an arithmetic sequence also.

Exercises on the arithmetic means

34 Let the arithmetic mean of two numbers be 11 and the arithmetic mean of their squares be 125 , so find the two numbers. « 9 , 13 »

35 Insert 16 arithmetic means between 27 and – 24 « (24 , 21 , ... , – 21) »

36 Insert 8 arithmetic means between log 2 and log 1024 « (2 log 2 , 3 log 2 , ... , 9 log 2) »

37 Find the number of the arithmetic means which are inserted between 1 and 17 and the seventh mean equals three times the second mean. « 7 »

38 Find the arithmetic sequence whose ninth term = 25 and the arithmetic mean between the third and fifth terms is 10 « (1 , 4 , 7 , ...) »

39 Find the arithmetic sequence in which the arithmetic mean between its third and seventh terms is 19 and its tenth term is greater than twice of its fourth term by 2 « (7 , 10 , 13 , ...) »

40 Find the arithmetic sequence whose sum of its second and fourth means equals 12 and the seventh mean is greater than the third mean by 4 « (3 , 4 , 5 , ...) »

41 If we insert a number of arithmetic means between 2 and 47 , then we find the ratio between the second mean and the last mean is 2 : 11 , find the number of means. « 14 »

42 Some arithmetic means are inserted between 20 and 170 , if the sum of the fifteenth and the twentieth means equals 5 times the fifth mean , find the number of the inserted means. « 24 »

43 If we insert n arithmetic means between 6 and 36 such that the ratio between the sum of the first two means and the sum of the last two means is 1 : 3 , then find n « 9 »

44 **Creative thinking :** If ℓ and m are two arithmetic means between x and y where $\ell > m$, prove that : $\ell - m = \frac{1}{3}(x - y)$

45 If a , b , c in an arithmetic sequence , **prove that :** $\left(\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}\right)$ are also in an arithmetic sequence.

Third Higher skills

1 **Choose the correct answer from those given :**

(1) If the last term of an arithmetic sequence is 10 times its first term and the term before the last one equals the sum of its fourth anf fifth term , then $\frac{T_6}{T_3}$ = ...

(a) 3 d (b) 2 (c) 2 d (d) $\frac{5}{2}$

(2) If d_1 is the common difference of the arithmetic sequence $(a, x_1, x_2, \ldots, x_5, b)$ and d_2 is the common difference of the arithmetic sequence $(a, y_1, y_2, \ldots, y_7, b)$, then $\frac{d_1}{d_2}$ = ...

(a) $\frac{3}{2}$ (b) $\frac{5}{7}$ (c) $\frac{4}{3}$ (d) $\frac{7}{5}$

(3) If x , y , z are three consecutive terms from an arithmetic sequence , then $(x + 2y - z)(2y + z - x)(z + x - y)$ = ...

(a) $\frac{1}{2} x y z$ (b) $x y z$ (c) $2 x y z$ (d) $4 x y z$

(4) If the volume of a cuboid equals 105 cm^3 and its dimensions form an arithmetic sequence and the sum of its dimensions is 15 cm. , then the length of its greatest dimension equals ..... cm.

(a) 5 (b) 6 (c) 7 (d) 8

(5) If (a , b , c , f , e) is an arithmetic sequence then a – 4 b + 6 c – 4 f + e = ...

(a) b + c (b) f – b (c) zero (d) 3

(6) If $\left(\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}\right)$ is an arithmetic sequence , then which of the following is an arithmetic sequence too ?

(a) b^2, c^2, a^2 (b) a^2, b^2, c^2 (c) c^2, a^2, b^2 (d) b^2, a^2, c^2

(7) The terms of an arithmetic sequence are integers and its common difference d where $2 < d < 7$ if one of its terms equals 115 , and another term equals 166 , then its common difference = ...

(a) 3 (b) 4 (c) 5 (d) 6

(8) If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 \left(2^x - \frac{7}{2}\right)$ form an arithmetic sequence , then x = ...

(a) 2 (b) 3 (c) 4 (d) 5

(9) If $\frac{a^{n+2} + b^{n+2}}{a^{n+1} + b^{n+1}}$ is an arithmetic mean between a and b , then n = ...

(a) – 1 (b) 1 (c) – 2 (d) zero

(10) The number of common terms between the two arithmetic sequences (3 , 7 , 11 , ... , 407) and (2 , 9 , 16 , ... , 709) equals .....

(a) 14 (b) 21 (c) 28 (d) 35

---

2 (T_n) is an arithmetic sequence where $T_n \neq$ zero **Prove that :**

(1) $\frac{1}{T_1 T_2} + \frac{1}{T_2 T_3} + \cdots + \frac{1}{T_{n-1} T_n} = \frac{n-1}{T_1 T_n}$

(2) $\frac{1}{\sqrt{T_1} + \sqrt{T_2}} + \frac{1}{\sqrt{T_2} + \sqrt{T_3}} + \cdots + \frac{1}{\sqrt{T_{n-1}} + \sqrt{T_n}} = \frac{n-1}{\sqrt{T_1} + \sqrt{T_n}}$

Applications on the arithmetic sequence

1 📖 **Physics :** Kareem has started to ride his motorcycle from the highest point of a downhill. He covered a distance of 100 cm. in the first second. In each next second , he was covering a distance that increases in a magnitude of 120 cm. from the directly previous distance. Find the distance which he covers in the tenth second. « 1180 cm. »

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2 📖 **Trade :** A man has bought a motorcycle and he has agreed with the seller to pay its price in monthly installments forming an arithmetic sequence whose n^{th} term is 120 n + 80 If the last installment was 1400 L.E. , find the number of the installments. « 11 »

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3 **Geometry :** ABC is a triangle in which the measure of one of its angles is the arithmetic mean of the measures of the other two angles. Let the difference between the measures of the smallest and the largest angles equal 80° Find the measure of each angle.

« 20° , 60° , 100° »

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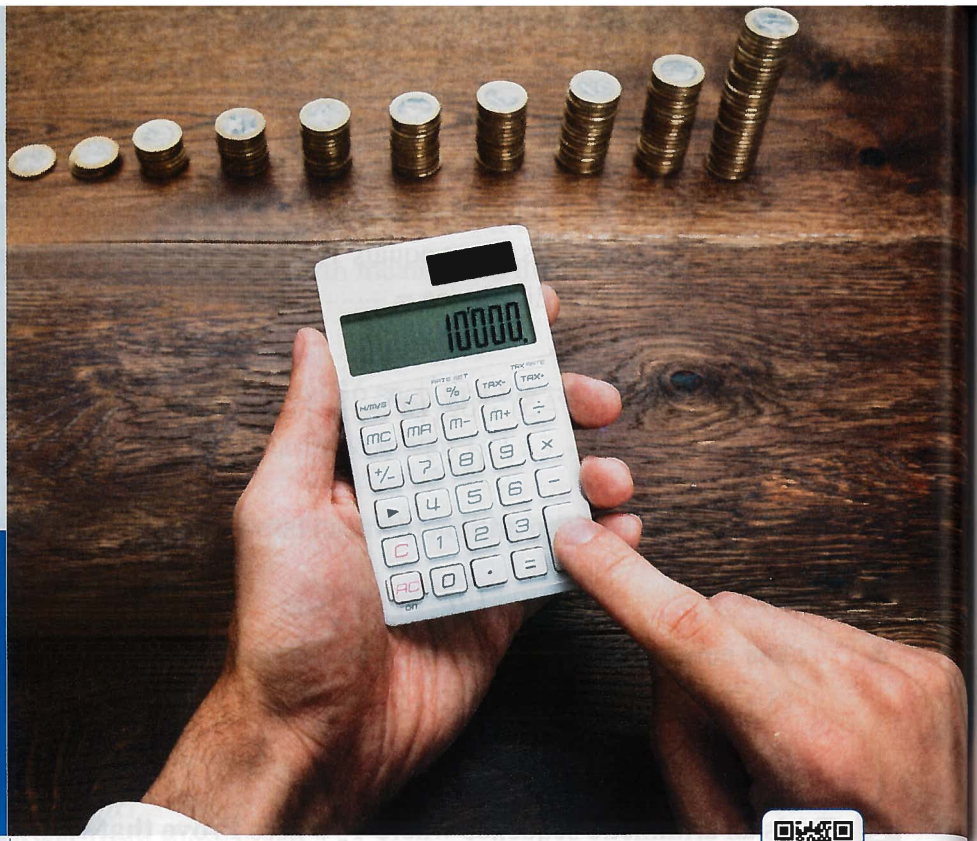
4 **Geometry :** Find the ratio among the lengths of sides of Δ ABC which is right-angled at B and a is the arithmetic mean of b and c « 4 : 5 : 3 »

Exercise

4

Arithmetic series



From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The value of the arithmetic series $\sum_{r=1}^{5} (2r + 1)$ equals .....

(a) 25 (b) 30 (c) 35 (d) 40

(2) The value of the arithmetic series $\sum_{r=1}^{20} (3 - 2r)$ equals .....

(a) 400 (b) – 400 (c) – 360 (d) 380

(3) The value of the series $4 + 9 + 14 + ... + (5n - 1)$ by using the summation notation equals .....

(a) $\sum_{r=4}^{n} (5r - 1)$ (b) $\sum_{r=1}^{n} (5r + 1)$ (c) $\sum_{r=1}^{n} (5r - 1)$ (d) $\sum_{r=1}^{5n-1} (3r + 1)$

(4) The value of the series $7 + 12 + 17 + 22$ by using the summation notation equals .....

(a) $\sum_{r=1}^{4} (5r + 2)$ (b) $\sum_{r=1}^{4} (4r + 3)$ (c) $\sum_{r=1}^{4} (7r + 1)$ (d) $\sum_{r=1}^{4} (3r + 4)$

(5) The sum of the terms of the arithmetic sequence $(3 , 5 , 7 , ... , (2n + 1))$ starting from its first term equals .....

(a) $n(n + 1)$ (b) $n(n + 2)$ (c) $n(n + 5)$ (d) $n(n + 2)(n + 3)$

(6) The sum of the first term and last term of an arithmetic sequence is 46 and the sum of all its terms is 345 , then the number of its terms is .....

(a) 25 (b) 15 (c) 23 (d) 22

(7) The first term of an arithmetic sequence is 12 and its last term is – 26 and the sum of its terms equals – 140 , then the sequence is .....

(a) (12 , 8 , 4 , ... , – 26) (b) (12 , 9 , 6 , ... , 26)

(c) (12 , 6 , 0 , ... , – 26) (d) (12 , 10 , 8 , ... , – 26)

(8) The sum of n terms from the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+...$ equals .....

(a) $\frac{n(n+1)}{2}$ (b) $2n(n+1)$ (c) $\frac{n(n+1)}{\sqrt{2}}$ (d) 1

(9) The sum of terms in the arithmetic series $89 + 85 + 81 + ... + 33 =$ .....

(a) 900 (b) 910 (c) 895 (d) 915

(10) The sum of terms in the arithmetic series $\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+...+\frac{13}{2} =$ .....

(a) 23.5 (b) 24.5 (c) 25 (d) 22.5

(11) 📖 The sum of terms in the arithmetic series $2 + 5 + 8 + ... + 62 =$ .....

(a) 664 (b) 670 (c) 660 (d) 672

(12) The sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+...$ to 9 terms = ...

(a) $\frac{-5}{6}$ (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{-3}{2}$

(13) 📖 The sum of the first 30 terms from the sequence (T_n) where $T_n = 2n + 3$ is .....

(a) 1000 (b) 1024 (c) 1020 (d) 1010

(14) The sum of the first 10 even natural numbers equals .....

(a) 20 (b) 45 (c) 55 (d) 90

(15) The sum of the first n terms from the sequence of the natural odd number is .....

(a) $2n$ (b) n^2 (c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

(16) 📖 The sum of natural numbers divisible by 3 between 30 and 50 equals .....

(a) 81 (b) 243 (c) 343 (d) 512

(17) The sum of the integers included between 100 and 170 and each of them is not divisible by 3 equals .....

(a) 5150 (b) 7120 (c) 6170 (d) 6210

(18) 📖 The sum of odd natural numbers more than 10 and less than 30 equals .....

(a) 100 (b) 150 (c) 200 (d) 250

(19) The sum of integers from 1 to 100 divisible by 2 or 5 equals .....

(a) 3000 (b) 3010 (c) 3150 (d) 3050

(20) The sum of two digit odd numbers equals .....

(a) 2475 (b) 2530 (c) 4905 (d) 5049

(21) For all the values of $n \in \mathbb{Z}^+$, if $a = 1 + 3 + 5 + ... + (2n - 1)$, then a is .....
(a) an even number. (b) perfect cube number.
(c) an odd number. (d) perfect square number.

(22) The sum of 30 consecutive terms of the sequence $(2n - 1)$ starting from T_{15} is .....
(a) 1335 (b) 1740 (c) 1675 (d) 1720

(23) The sum of the first 8 terms of the arithmetic sequence whose fourth term is 2 and its seventh term is $\frac{1}{4}$ is .....
(a) 13 (b) $12\frac{1}{3}$ (c) $13\frac{2}{3}$ (d) $13\frac{1}{2}$

(24) Which of the following sequences the sum of its first 20 terms equals 820 ?
(a) (2 , 6 , 10 , ...) (b) (1 , 5 , 9 , ...) (c) (3 , 7 , 11 , ...) (d) (4 , 8 , 12 , ...)

(25) If the sum of the first twenty terms from an arithmetic sequence equals 860 and the sum of its third term and fourth term is more than the sixth term by 5 , then the sequence is .....
(a) (4 , 9 , 14 , ...) (b) $\left(5 , 9\frac{1}{2} , 14 , ...\right)$ (c) (5 , 9 , 13 , ...) (d) (5 , 8 , 11 , ...)

(26) The arithmetic sequence whose second term is 13 and the sum of its first 10 terms equals 235 is .....
(a) (8 , 13 , 18 , ...) (b) (9 , 13 , 17 , ...) (c) (12 , 13 , 14 , ...) (d) (10 , 13 , 16 , ...)

(27) If (T_n) is an arithmetic sequence in which $T_n = 81 - 2n$, then S_{80} starting from the first term = ...
(a) zero (b) 81 (c) – 81 (d) – 162

(28) The first term of an arithmetic sequence is 7 and the sum of its first 10 terms = 250 , then T_{20} = ...
(a) 87 (b) 83 (c) 86 (d) 91

(29) An arithmetic sequence consists of 27 terms and its middle term is 41 , then the sum of all terms of this sequence = ...
(a) 553.5 (b) 1107 (c) 2214 (d) 68

(30) The number of terms in an arithmetic sequence consists of n terms where n is an odd number and its middle term = m , then the sum of all terms of this sequence = ...
(a) 2 mn (b) $\frac{1}{2}$ mn (c) mn (d) mn^2

(31) The sum of even ordered terms from the arithmetic sequence (5 , 7 , 9 , ... , 65) is .....
(a) 520 (b) 530 (c) 525 (d) 560

(32) The sum of the last half terms from the arithmetic sequence (12 , 8 , 4 , ... , – 24) equals .....
(a) – 85 (b) – 80 (c) – 75 (d) – 70

(33) The sum of the last quarter terms from the arithmetic sequence (7 , 11 , 15 , ... , 163) equals .....

(a) 1500 (b) 1420 (c) 1450 (d) 1430

(34) The number of terms which must be taken from the arithmetic sequence (1 , 3 , 5 , ...) starting from its first term, such that their sum equals 400 is ..... terms.

(a) 18 (b) 21 (c) 20 (d) 19

(35) The number of the terms necessary to be taken from the sequence (27 , 24 , 21 , ...) starting from the first term to vanish the sum is ..... terms.

(a) 18 (b) 19 (c) 20 (d) 21

(36) If the sum of the first 20 terms in an arithmetic sequence is 190 , and the sum of the next ten terms equals 395 , then T_8 = ...

(a) $-9\frac{1}{2}$ (b) $-5\frac{1}{2}$ (c) $2\frac{1}{2}$ (d) $4\frac{1}{2}$

(37) If (T_n) is an arithmetic sequence , the number of its terms is (n) , $T_1 = 12$, $T_n = 78$, $S_n = 1035$, then T_4 = ...

(a) 15 (b) 18 (c) 21 (d) 24

(38) In an arithmetic sequence $T_3 = 5$, $T_{n-2} = 95$, the sum of its first n terms equals 1000 , then n = ...

(a) 10 (b) 12 (c) 16 (d) 20

(39) In the arithmetic sequence (1 , 3 , 5 , ...). A at which term you should start to get the sum of 10 consecutive terms equals 200 ?

(a) T_3 (b) T_6 (c) T_9 (d) T_{11}

(40) If S_n is the sum of the first n terms from an arithmetic sequence whose first term = a and its common difference = d , then $S_n - 2\,S_{n-1} + S_{n-2}$ = ... where $(n > 2)$

(a) 2 d (b) a (c) d (d) a + d

(41) The greatest number of terms can be taken from the sequence (T_n) = (32 , 28 , 24 , ...) starting from the first term to get a positive sum is .....

(a) 16 (b) 17 (c) 18 (d) 20

(42) The greatest sum of the terms of the sequence in which $T_6 = 16$, $T_{18} = -20$ is .....

(a) 173 (b) 176 (c) 179 (d) 182

(43) The greatest sum of the terms of the arithmetic sequence (33 , 31 , 29 , ...) equals .....

(a) 280 (b) 298 (c) 290 (d) 289

(44) The smallest sum of the terms of the arithmetic sequence (– 24 , – 20 , – 16 , ...) equals .....

(a) zero (b) – 86 (c) – 84 (d) – 88

(45) The least number of terms that can be taken from the sequence (89 , 81 , 73 , ...) starting from the first term to make the negative sum is ..... terms.

(a) 25 (b) 24 (c) 26 (d) 23

(46) The greatest number of terms that can be taken from the sequence (25 , 21 , 17 , ...) starting from the first term to make the positive sum is ..... terms.

(a) 12 (b) 14 (c) 13 (d) 11

(47) If the sum of the forty means between two numbers is 120 , then the sum of the fifty means between the same two numbers is .....

(a) 130 (b) 160

(c) 150 (d) nothing of the previous.

(48) When you insert 28 arithmetic means between 4 and 91 , then the sum of the terms of the arithmetic sequence resulted = ...

(a) 1225 (b) 1315 (c) 1425 (d) 1525

(49) When you insert 16 arithmetic means between a and b , then the sum of terms of the produced sequence equals .....

(a) 8 (a + b) (b) 9 (a + b) (c) 16 (a + b) (d) 18 (a + b)

(50) When n arithmetic means are inserted between 3 and 51 , the sum of the resulted sequence equals .....

(a) 27 (n – 2) (b) 27 (n – 1) (c) 27 (n + 1) (d) 27 (n + 2)

(51) The arithmetic sequence in which the sum of its first 20 terms equals 820 and the arithmetic mean of its T_4 and T_7 equals 21 is .....

(a) (2 , 7 , 12 , ...) (b) (4 , 8 , 12 , ...) (c) (5 , 7 , 9 , ...) (d) (3 , 7 , 11 , ...)

(52) In the arithmetic sequence (5 , 8 , 11 , ...) , then :

First : The sum of its first twenty terms = ...

(a) 670 (b) 820 (c) 525 (d) 690

Second : The sum of ten terms starting from the seventh term = ...

(a) 375 (b) 385 (c) 355 (d) 365

Third : The sum of the sequence terms starting from T_{10} up to T_{20} = ...

(a) 513 (b) 510 (c) 517 (d) 520

(53) The arithmetic sequence in which its second term is 23 and its term before the last one equals 97 and the sum of its terms equals 2400 , then :

First : Number of its terms = ... terms.

(a) 38 (b) 40 (c) 42 (d) 43

Second : The sequence is .....

(a) (19 , 23 , 27 , ... , 98) (b) (18 , 23 , 28 , ... , 108)

(c) (21 , 23 , 25 , ... , 99) (d) (20 , 23 , 26 , ... , 102)

(54) If the sum of the first n terms from an arithmetic sequence is identified by the rule : $S_n = 2n(7 - n)$, then :

First : $T_7 =$ .....

(a) – 15 (b) – 12 (c) – 14 (d) – 11

Second : The number of terms necessary to be taken from the sequence starting from the first term to make the sum equal to – 240 is ..... terms.

(a) 15 (b) 17 (c) 14 (d) 12

(55) In an arithmetic sequence , $S_5 - S_4 = 20$, $S_8 - S_7 = 29$, then T_{51} = ...

(a) 49 (b) 98 (c) 155 (d) 158

(56) In an arithmetic sequence , the first term = 3 and its last term = 39 , the sum of its terms = 210 , then the number of its terms = ...

(a) 8 (b) 10 (c) 12 (d) 15

(57) The sum of first 5 terms of an arithmetic sequence = 10 and the sum of its first 10 terms = 5 , then the sum of its first 15 terms = ...

(a) 15 (b) – 15 (c) 50 (d) – 50

(58) In an arithmetic sequence $T_1 = 10$, $T_m = x$, $T_x = m$ and S_n = zero , then n = ...

(a) $x + m$ (b) $x - m$ (c) 20 (d) 21

(59) If the sum of the first n terms of a sequence is $\frac{n}{n+1}$, then $\frac{1}{T_8}$ = ...

(a) 64 (b) 80 (c) 75 (d) 72

(60) If $\frac{3 + 5 + 7 + \text{... to n terms}}{5 + 8 + 11 + \text{... to 10 terms}} = 7$, then n = ...

(a) 35 (b) 36 (c) 37 (d) 40

(61) $\frac{2 + 6 + 10 + \text{... to 75 terms from an arithmetic sequence}}{3 + 9 + 15 + \text{...} + 297}$ = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

(62) $\frac{1 + 3 + 5 + \text{... to n terms from an arithmetic sequence}}{2 + 4 + 6 + \text{... to n terms from an arithmetic sequence}}$ = ...

(a) $\frac{2n}{1+n}$ (b) $\frac{n+1}{2}$ (c) $\frac{n}{n+1}$ (d) $\frac{n+1}{n+2}$

(63) The arithmetic sequence in which $T_4 = 24$ and the ratio between the sum of the first five terms to the sum of the next five terms directly is 1 : 2 is .....

(a) (18 , 20 , 22 , ...) (b) (15 , 18 , 21 , ...)

(c) (12 , 16 , 20 , ...) (d) (9 , 14 , 19 , ...)

(64) What is the number of clock chimes in one day if you know that it is chiming once at one o'clock , twice at two o'clock and so on ?

(a) 156 (b) 132 (c) 120 (d) 78

(65) If S_n is the sum of the first n terms from an arithmetic sequence , $S_1 = 6$, $S_7 = 105$, then $\frac{S_n}{S_{n-3}}$ = ... where $n > 3$

(a) $\frac{n}{n-3}$ (b) $\frac{n+3}{n}$ (c) $\frac{n+3}{n-3}$ (d) $\frac{n}{n+3}$

(66) If $\sum_{r=1}^{n} 1 + \sum_{r=1}^{n} 2 + \sum_{r=1}^{n} 3 + ... + \sum_{r=1}^{n} 20 = 1050$, then n = ...

(a) 2 (b) 3 (c) 4 (d) 5

(67) If (0 , 1 , 2 , 3 , ...) is an arithmetic sequence where S_m is the sum of the first m terms from the odd - ordered terms , S_n is the sum of the first n terms from the even - ordered terms , then $\frac{S_m}{S_n}$ = ...

(a) $\frac{m^2-1}{n^2}$ (b) $\frac{m^2-m}{n^2}$ (c) $\frac{m(m+1)}{n^2-1}$ (d) $\frac{m^2-m}{n^2+n}$

(68) The number of terms of an arithmetic sequence is (2 n + 1) , then the ratio between the sum of odd orders to the sum of even orders is .....

(a) $\frac{n}{n+1}$ (b) $\frac{n+1}{n}$ (c) $\frac{n^2}{n+1}$ (d) $\frac{n+1}{2n}$

(69) An arithmetic sequence consists of 99 terms , the sum of odd orders equals 2550 , so the sum of all terms equals .....

(a) 5049 (b) 5050 (c) 5100 (d) 5409

(70) An arithmetic sequence consists of 51 terms , the sum of odd orders : the sum of all terms = ...

(a) 50 : 51 (b) 25 : 26 (c) 25 : 51 (d) 26 : 51

(71) A student was calculating the common difference of an arithmetic sequence (its first term is known) the student thought that the common difference was – 2 , but the right one is 2. If the sum of the first five terms in the wrong arithmetic sequence was – 5 , then the sum of the first five terms in the right one is .....

(a) 25 (b) – 25 (c) – 35 (d) 35

(72) If $S = \sum_{r=1}^{n} (2r + 1)$, $M = \sum_{r=1}^{n} (3r)$, then ..... (where $n > 1$)

(a) $S > M$ (b) $S < M$

(c) $S = M$ (d) we cannot compare between them

(73) Seats are organized in a theatre such that the last row contains 45 seats , then each next row is less than the previous row by 2 seats. If the number of seats in the theatre is 520 seats , then the number of rows = ...

(a) 20 (b) 26 (c) 20 or 26 (d) 23

(74) A straight road of length 110 km. Two cars start to move on it in opposite direction , one from each end , the first covers 8 km. in the first hour and decreases this distance by $\frac{1}{2}$ km. each consecutive hour , the second car covers 4 km. in the first hour and increases this distance by 1 km. each consecutive hour , then the time taken till the two cars meet is ..... hours.

(a) 5 (b) 8 (c) 10 (d) 12

Second Essay questions

1 **Find :**

(1) The sum of the terms of the arithmetic sequence (2 , 5 , 8 , ... , 80) « 1107 »

(2) The sum of the integers included between 3 and 1000 and each of them is divisible by 7 « 71071 »

(3) The sum of the odd ordered terms of the arithmetic sequence (2 , 5 , 8 , ... , 110) « 1064 »

(4) The sum of the last half of the terms of the arithmetic sequence (8 , 11 , 14 , ... , 71) « 616 »

(5) The sum of the last third of the terms of the arithmetic sequence (25 , 21 , 17 , ... , – 127) « – 1339 »

2 How many terms must be taken from the sequence (40 , 36 , 32 , ...) starting from its first term such that their sum becomes 208 ? Explain the two answers. « 8 or 13 »

3 If (– 115 , – 109 , – 103 , ...) is an arithmetic sequence , **find :**

(1) The order of the first positive term.

(2) The least number of its terms starting from the first one to give a positive sum. « 21 , 40 »

4 Find the order of the first negative term of the terms of the sequence (152 – 9 n), then find the greatest sum can be got from the terms of this sequence. « 17 , 1208 »

5 **Find the arithmetic sequence in which :**

(1) $T_1 = 23$, $T_n = 86$ and $S_n = 545$ « (23 , 30 , 37 , ... , 86) »

(2) $T_1 = 17$, $T_n = -95$ and $S_n = -585$ « (17 , 9 , 1 , ... , – 95) »

6 Insert 17 arithmetic means between 42 and – 12 , then find the order of the first negative term and the sum of the sequence terms. « 16 , 285 »

7 Find the arithmetic sequence whose sum of its third and fifth terms is 22 and its fourth term is less than its seventh term by 9, then find the sum of the first twenty five terms. « (2 , 5 , 8 , ...) , 950 »

8 An arithmetic sequence whose sum of its first and last terms is 26 and the sum of its terms is 468 , find the number of its terms. If its tenth term equals 47 , find this sequence. « 36 , (83 , 79 , 75 , ...) »

9 An arithmetic sequence in which $T_1 + T_2 = 222$ and the sum of its first 10 terms is 1030, find the sequence , then find the least number of its terms that have to be taken starting from its first term to make the sum negative. « (112 , 110 , 108 , ...) , 114 »

10 An arithmetic sequence in which its first term is 29 and its second term equals 5 times its seventh term. Find the sequence and the number of terms which must be taken starting from its first term to make its sum maximum. « (29 , 25 , 21 , ...) , 8 »

11 In an arithmetic sequence , $T_{20} = 41$, and the sum of the third term and the sixth term exceeds the ninth term by 1 , find the sequence and the number of terms must be taken starting from the first term to make the sum 440 « (3 , 5 , 7 , ...) , 20 »

12 What is the arithmetic sequence whose first term is greater than twice its fifth term by 2 and the arithmetic mean of the third and sixth terms is 16 ? How many terms should be taken starting from the first term to make the sum equal to zero ? « (30 , 26 , 22 , ...) , 16 »

13 Find the arithmetic sequence whose terms are positive , the sum of its first three terms equals 30 and the product of its third and fourth terms equals 300 , then find the sum of its first fifteen terms. « (5 , 10 , 15 , ...) , 600 »

14 If the sum of the first eleven terms of an arithmetic sequence is 308 and the product of its second and sixth terms is 224 , find the sequence. « (3 , 8 , 13 , ...) »

15 The sum of the third and the fifth terms in an increasing arithmetic sequence equals 24 , and the square of its sixth term equals 324 , find the sequence and the sum of the first 20 terms. « (3 , 6 , 9 , ...) , 630 »

16 An arithmetic sequence in which $T_{36} = 0$. If the sum of the first n terms = twice the sum of its first five terms , find the value of n « 11 or 60 »

17 If n arithmetic means are inserted between 1 and 31 and the ratio of the seventh mean to the last mean is $\frac{15}{29}$, find n and the sum of the sequence. « 14 , 256 »

18 Find the arithmetic sequence in which the sum of its first seven terms equals 245 and the sum of its next seven terms equals 98 « (44 , 41 , 38 , ...) »

19 An arithmetic sequence has a common difference which is 2 , find this arithmetic sequence such that the sum of its first n terms equals 320 and the sum of its first 2 n terms equals1152 Find also the number of terms which must be taken , starting from its first term , such that their sum equals 725 « (5 , 7 , 9 , ...) , 25 »

20 How many terms must be taken starting from the first term from the sequence $(T_n) = (4n + 2)$ such that the sum of the last third of its terms equals four times the sum of the first third of its terms ? « 18 »

21 An arithmetic sequence is composed of 33 terms. The sum of its first eleven terms equals 264 , the sum of its last eleven terms equlas 330 , find the sum of this sequence , then find the sum of its five middle terms.

« 891 , 135 »

22 **Discover the error :**

(1) To find the greatest sum of an arithmetic sequence, we find the number of its positive terms by putting $S_n > 0$ to find the value of n , and then we can find the greatest sum.

(2) To find the least sum of an arithmetic sequence, we find the number of its negative terms by putting $S_n < 0$ to find the value of n , and then we can find the least sum.

(3) To find the number of the terms of an arithmetic sequence at which the sum is vanished we put $S_n = 0$, then $a + (n - 1)\,d = 0$ [where $n \neq 0$] , then we find the number of terms.

(4) If the sum of the first n terms of the terms of an arithmetic sequence is given by the relation $S_n = \frac{n}{2}(3n + 5)$ then $T_n = S_{n+1} - S_n$

Third Higher skills

Choose the correct answer from those given :

(1) If n arithmetic means are inserted between two numbers a and b , then the sum of these means equals .....

(a) $\frac{a+b}{2}$ (b) $n\left(\frac{a+b}{2}\right)$ (c) $n(b - a)$ (d) $\frac{n}{2}(b - a)$

(2) If (T_n) is an arithmetic sequence in which $T_1 + T_5 + T_{10} + T_{16} = 64$, then the sum of the first 15 terms = ...

(a) 120 (b) 180 (c) 240 (d) 360

(3) If $S_n = S_m$ from an arithmetic sequence where $n \neq m$, then S_{n+m} = ...

(a) $m + n$ (b) $-(m + n)$ (c) zero (d) $m^2 + n^2$

(4) If the measures of interior angles in a polygon form an arithmetic sequence and the measure of the smallest angle in the polygon = 120° and the common difference of the sequence = 5° , then the number of its sides = ...

(a) 3 or 9 (b) 4 or 16 (c) 3 or 4 (d) 9 or 16

(5) In an arithmetic sequence $\frac{S_m}{S_n} = \frac{m^2}{n^2}$, then $\frac{T_m}{T_n}$ = ... where $m \neq n$

(a) $\frac{m-1}{n-1}$ (b) $\frac{n-1}{m-1}$ (c) $\frac{2m-1}{2n-1}$ (d) $\frac{2n-1}{2m-1}$

(6) If (T_n) is an arithmetic sequence , then the expression $(T_1^2 - T_2^2 + T_3^2 - T_4^2 + T_5^2 - T_6^2 + ... + T_{2n-1}^2 - T_{2n}^2)$ equals .....

(a) $-nS_{2n}$ (b) $-dS_{2n}$ (c) $S_{2n} - S_n$ (d) $S_{2n}^2 - S_n^2$

(7) If S_n is the sum of the first n terms of an arithmetic sequence and $S_{2n} = 3\ S_n$, then $S_{3n} : S_n =$ .....

(a) 4 (b) 6 (c) 8 (d) 10

(8) T_{99} in the series $2 + 7 + 14 + 23 + 34 + ...$ is .....

(a) 9999 (b) 9998

(c) 10000 (d) nothing of the previous.

(9) If $(\log x , \log x\,y , \log x\,y^2 , ...)$ is an arithmetic sequence and $x = 160$, $y = \frac{1}{2}$, then $S_9 =$ .....

(a) 9 (b) 9 log 25 (c) 25 log 10 (d) 18 log 16

(10) If $T_1 = 5$ in the sequence (T_n) where $T_n - T_{n-1} = 2n + 3$ for each $n \geq 2$, then $T_{50} =$ .....

(a) 2700 (b) 2702 (c) 2704 (d) 2706

Applications on the arithmetic sequence

1 A 25 - row theatre ; the first row contains 20 seats , the second row contains 22 seats and the third row contains 24 seats and so on ... Find the number of seats in all the theatre rows. « 1100 seats »

2 **Saving :** Zyad saves L.E. 15 from his daily work. If he saves an amount of money increasing for L.E. 2 every day more than the day before directly. Find the sum of what he can save within 15 days. « L.E. 435 »

3 **Tarde :** A man has borrowed an amount of money , he has handled to pay it back over 10 installments. How much is the loan if the first installment is L.E. 500 and each next installment is more than the directly previous installment for L.E. 200 « L.E. 14000 »

4 A man owes L.E. 48000 , he decided to repay his debt on twenty installments yearly forming an arithmetic sequence and after he paid five installments , he died and owed $\frac{4}{5}$ of the debt. What was the first installment ? « 1792 »

5 21 fruits are put on a straight line. The distance between any two fruits is 2 metres. A basket is put beside the first one. A man collected the fruits one by one and put it in the basket without moving it. Find the total distance he walked. « 840 m. »

6 **Sports :** Kareem is preparing himself for a long-distance race. He has decided to run 4 km. on the first day , then increased the distance for half a kilometer daily.

(1) Find the distance , which Kareem covers on the seventh day.

(2) Find the sum of the distances , which Kareem covers in the first week (A week is seven days).

(3) How many days does Kareem need to cover 81 km. if he continues to exercise according to this pattern ?

« 7 km. , $38\frac{1}{2}$ km. , 12 days »

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7 **Physics :** An object fell from a height of 490 m. under the effect of the gravity. Disregarding the air resistance, it covers a distance of 4.9 m. in the first second, 14.7m. in the second second and 24.5m. in the third second and so on... Find :

(1) The distance covered by the object in the sixth second.

(2) The sum of distances covered in the first eight seconds.

(3) When does the object hit the ground ?

« 53.9 m. , 313.6 m. , 10 seconds »

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8 A customer deposits a fixed amount of money at the beginning of each month in a bank which gives 10 % simple interest per year. The total interest at the end of the year became L.E. 117. Calculate the monthly deposit.

« L.E. 180 »

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9 A man bought a flat whose price is L.E. 164000 He paid L.E. 68000 as a down payment and paid the rest of the price in monthly installments which form an arithmetic sequence whose n^{th} term is 400 n + 600 , find the number of installments.

« 20 »

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10 **Activity :**

Kareem has a store for food commodities. He arranges the tuna cans in rows so that he puts 12 cans in the (first) lowest row, 11 in the next row and 10 in the next and so on...

(1) Find the number of the tuna cans in the seventh row.

(2) In which row are the tuna cans three cans ?

(3) Find the number of the tuna cans starting from the first row up to the last row containing one tuna can.

« 6 , 10 , 78 »

Exercise

5

Geometric sequence

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from those given :

Exercises on the definition of the geometric sequence , its general term and identifying the geometric sequence

(1) The fifth term from the sequence (T_n) when $T_n = 2 \times (3)^{n-1}$ equals .....

(a) 81 (b) 162 (c) 324 (d) 243

(2) The fourth term in the geometric sequence $\left(\sqrt[4]{3}, \sqrt[8]{3}, 1, ...\right)$ is .....

(a) $\sqrt[16]{3}$ (b) $\frac{1}{\sqrt[4]{3}}$ (c) $\frac{1}{\sqrt[8]{3}}$ (d) $\sqrt[32]{3}$

(3) In the geometric mean $(T_n) = (-2560, 1280, ..., -10)$, then the fourth term from the end = ...

(a) 40 (b) – 40 (c) 80 (d) – 80

(4) Which of the following is a geometric sequence ?

(a) (2 , 5 , 8 , 11 , ...) (b) (3 , 33 , 3 , 33 , ...)

(c) (5 , – 5 , 5 , – 5 , ...) (d) (1 , 4 , 8 , 16 , 32 , ...)

(5) All the following sequences are geometric except .....

(a) (3 , – 6 , 12 , – 24 , ...) (b) $(\log a, \log a^2, \log a^3, \log a^4, ...)$

(c) $\left(\frac{3}{2}, 1, \frac{2}{3}, \frac{4}{9}\right)$ (d) $\left(\frac{3b}{2a}, 3, \frac{6a}{b}, \frac{12a^2}{b^2}\right)$

(6) Of all the following sequences , the geometric sequence is .....

(a) $(T_n) = (4\ n^2)$ for each $n \geq 1$ (b) $(T_n) = \left(\frac{1}{4} \times T_{n-1}\right)$ for each $n \geq 2$, $T_1 = 1$

(c) $(T_n) = (2^n - 1)$ for each $n \geq 1$ (d) $(T_n) = \left(\log (3 \times 2^n)\right)$ for each $n \geq 1$

(7) The common ratio of a geometric sequence $= \frac{1}{2}$ and its third term is 24 , then the sequence is .....

(a) (48 , 24 , 12 , ...) (b) (6 , 12 , 24 , ...) (c) (96 , 48 , 24 , ...) (d) $\left(\frac{8}{3}, 8, 24, ...\right)$

(8) The first term in a geometric sequence is 2 and its sixth term = 64 , then the sequence is .....

(a) (2 , 8 , 32, ...) (b) (2 , 4 , 8 , ...) (c) (2 , – 4 , 8 , ...) (d) (2 , 6 , 18 , ...)

(9) The sequence $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \sqrt{3}, ...\right)$ is .....

(a) a geometric , its common ratio = 2 (b) a geometric , its common ratio $= \frac{1}{\sqrt{3}}$

(c) an airthmetic , its common difference = 1 (d) an airthmetic , its common difference $= \frac{1}{\sqrt{3}}$

(10) The sequence $(15, 5, \frac{5}{3}, ...)$ is .....

(a) arithmetic and its common difference is – 5 (b) geometric and its common ratio is 3

(c) airthmetic and its common difference is 5 (d) geometric and its common ratio is 3^{-1}

(11) The sequence $\left(\frac{1}{243}, -\frac{1}{81}, \frac{1}{27}, -\frac{1}{9}, \frac{1}{3}\right)$ is .....

(1) a finite sequence. (2) an increasing sequence.

(3) an alternating sequence.

(4) a geometric sequence , its common ratio is – 3

(a) (1) only. (b) (1) and (2) only.

(c) (3) and (4) only. (d) (1) , (3) and (4) only.

(12) The n^{th} term in the geometric sequence (3 , – 6 , 12 , ...) is .....

(a) $3\ (-2)^{n-1}$ (b) $3\ (-2)^n$ (c) $(-6)^{n-1}$ (d) $(-6)^n$

(13) The n^{th} term of the geometric sequence $\left(\frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, ...\right)$ is .....

(a) $\left(\frac{-1}{2}\right)^{n-1}$ (b) $\left(\frac{1}{2}\right)^{n-1}$ (c) $\left(\frac{1}{2}\right)^n$ (d) $\left(\frac{-1}{2}\right)^n$

(14) The n^{th} term in the geometric sequence $(x^3, x, x^{-1}, ...)$ is .....

(a) x^{5-2n} (b) x^{5+n} (c) x^{n-5} (d) x^{n-1}

(15) If (– 1 , – 5 , – 25 , – 125 , ...) is a geometric sequence , then ..... for each $n > 1$

(a) $T_n = \frac{1}{5} T_{n-1}$ (b) $T_n = -5\ T_{n-1}$ (c) $T_n = 10\ T_{n-1}$ (d) $T_n = 5\ T_{n-1}$

(16) The next term of the geometric sequence $(8 , 6 , \frac{9}{2} , \frac{27}{8} , ...)$ is .....

(a) $\frac{11}{8}$ (b) $\frac{27}{16}$ (c) $\frac{9}{4}$ (d) $\frac{81}{32}$

(17) If $x > 0$, then the common ratio of the geometric sequence $(4 , x - 3 , 2x + 6 , ...)$ is .....

(a) 1 (b) 5 (c) 3 (d) 24

(18) If $(a , b , c , ...)$ is a geometric sequence its common ratio is r , then $\left(\frac{1}{a^2} , \frac{1}{b^2} , \frac{1}{c^2} , ...\right)$ represents a geometric sequence whose common ratio = ...

(a) r (b) $\frac{1}{r}$ (c) r^2 (d) $\frac{1}{r^2}$

(19) If a , b , c , d , e form a geometric sequence , then $\frac{e}{c}$ = ...

(a) $\frac{d}{c}$ (b) $\frac{b}{a}$ (c) $\frac{c}{b}$ (d) $\frac{d}{b}$

(20) The geometric sequence whose first term = a and its common ratio = r is increasing if .....

(a) $a > 0 , -1 < r < 0$ (b) $a > 0 , 0 < r < 1$ (c) $a < 0 , -1 < r < 0$ (d) $a < 0 , 0 < r < 1$

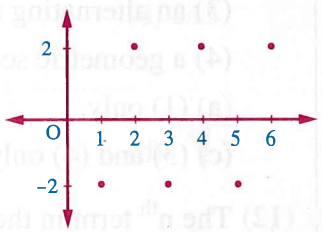
(21) The geometric sequence whose first term = a and its common ratio = r is decreasing if .....

(a) $a > 0 , -1 < r < 0$ (b) $a > 0 , 0 < r < 1$ (c) $a < 0 , -1 < r < 0$ (d) $a < 0 , 0 < r < 1$

(22) The geometric sequence whose first term = a and its common ratio = r is not alternating sign if .....

(a) $a > 0 , r < -1$ (b) $a > 0 , -1 < r < 0$ (c) $a > 0 , 0 < r < 1$ (d) $a < 0 , -1 < r < 0$

(23) The given figure represents a geomteric sequence its general term T_n = ...



(a) $2 \times (-1)^n$ (b) $(2)^n$

(c) $2 \times (-1)^{n+1}$ (d) $(-2)^n$

(24) If a , b , c is a geometric sequence its common ratio is r , then all the following statements are true except .....

(a) $r = \frac{b}{a}$ (b) $r = \frac{c}{b}$ (c) $r = \frac{b^2}{ac}$ (d) $r = \frac{b + c}{a + b}$

(25) If a , b , c are positive real numbers form a geometric sequence , then $(\log a , \log b , \log c)$ is .....

(a) a geometric sequence , its common ratio is $\frac{b}{a}$

(b) a geometric sequence , its common ratio $\log \frac{b}{a}$

(c) an arithmetic sequence , its common difference is $(b - a)$

(d) an arithmetic sequence , its common difference is $\log \frac{b}{a}$

(26) The order of the term whose value $\frac{1}{243}$ from the geometric sequence (81 , 27 , 9 , ...) equals .....

(a) 5 (b) 7 (c) 9 (d) 10

(27) The order of the first term less than one in the geometric sequence (1024 , 512 , 256 , ...) is .....

(a) 7 (b) 10 (c) 12 (d) 14

(28) If the third term in a geometric sequence = 4 , then the product of the first five terms is .....

(a) 4^2 (b) 4^3 (c) 4^5 (d) 4^6

(29) The product of the n^{th} term from the beginning by the n^{th} term from the end in a geometric sequence equals .....

(a) the last term. (b) the first term.

(c) the product of the first term by last term.

(d) nothing of the previous.

(30) If the third term from a geometric sequence equals square its first term , and its second term = 8 , then its sixth term = ...

(a) 120 (b) 124 (c) 128 (d) 132

(31) The number of terms in the geometric sequence $\left(243 , 81 , 27 , \cdots , \frac{1}{9}\right)$ equals .....

(a) 6 (b) 7 (c) 8 (d) 9

(32) The order of the term whose value = 1024 in the geometric sequence $\left(\frac{1}{8} , \frac{1}{4} , \frac{1}{2} , 1 , ...\right)$ is .....

(a) 16 (b) 14 (c) 12 (d) 11

(33) In the geometric sequence (6 , 12 , 24 , ...) , the order of the first term greater than 200 is .....

(a) 7 (b) 8 (c) 6 (d) 9

(34) All terms of a geometric sequence are positive , $T_5 - T_4 = 36$, $T_4 - T_2 = 40$, then the sequence is .....

(a) (64 , 32 , 16 , ...) (b) $\left(\frac{64}{3} , 32 , 48 , ...\right)$

(c) (32 , 48 , 72 , ...) (d) (24 , 36 , 54 , ...)

(35) All terms of a geometric sequence are positive , $T_3 + T_4 = 6\,T_2$, $T_7 = 320$, then the sequence is .....

(a) (2 , 10 , 20 , ...) (b) (24 , 16 , 8 , ...)

(c) (3 , 15 , 75 , ...) (d) (5 , 10 , 20 , ...)

(36) The sum of the first three terms in a geometric sequence is 26. The sum of the next three terms is 702 , then the sequence is .....

(a) (3 , 6 , 12 , ...) (b) $\left(\frac{26}{7}, \frac{52}{7}, \frac{104}{7}, ...\right)$ (c) (2 , – 8 , 32 , ...) (d) (2 , 6 , 18 , ...)

(37) The third term in a geometric sequence is more than the second term by 3 and the sum of the square of the second term and square of the third term is 45 , if the first term is positive , then the sequence is .....

(a) (3 , 6 , 12 , ...) (b) $\left(\frac{2}{3}, 2, 6, ...\right)$ (c) $\left(\frac{3}{2}, 3, 6, ...\right)$ (d) (2 , 6 , 18 , ...)

(38) The number of terms in a geometric sequence is n , its first term is a and its last term is ℓ , then the product of all its terms = ...

(a) $(a \times \ell)^n$ (b) $(a \times \ell)^{2n}$ (c) $\left(\frac{\ell}{a}\right)^{\frac{n}{2}}$ (d) $(a \times \ell)^{\frac{n}{2}}$

(39) In a geometric sequence , $T_m \times T_n = T_k \times T_q$, then $\frac{m+n}{k+q}$ = ...

(a) 1 (b) 3 (c) – 1 (d) – 2

(40) If (a , b , c) is a geometric sequence , all of its terms are positive , then the curve $y = a x^2 + b x + c$ .....

(a) touches the x-axis. (b) intersects the x-axis at two distinct points.

(c) lies above the x-axis. (d) lies below the x-axis.

Exercises on the geometric means

(41) If a , b , c form a geometric sequence , then .....

(a) $bc = a^2$ (b) $ab = c^2$ (c) $b^2 = ac$ (d) $abc = 1$

(42) The geometric mean of the two numbers 4 , 16 is .....

(a) 10 (b) 64 (c) 8 (d) ± 8

(43) If 3 , x , 9 form a geometric sequence , then x = ...

(a) ± 6 (b) $3\sqrt{3}$ (c) $\pm 3\sqrt{3}$ (d) $-\sqrt{3}$

(44) If the geometric mean of the two numbers 9 , y is 15 , then y = ...

(a) 6 (b) 5 (c) 25 (d) 9

(45) The arithmetic mean of two distinct positive real numbers ..... their geometric mean.

(a) = (b) < (c) > (d) ≤

(46) If (T_n) is a geometric sequence where $T_n = 7 \times (3)^{n-1}$, then the geometric mean between T_3 and T_7 is .....

(a) ± 570 (b) ± 567 (c) ± 540 (d) ± 560

(47) If (1 , m , 4 m – 4) is a geometric sequence , then m = ...

(a) 2 (b) – 2 (c) 4 (d) – 4

(48) Which of the following is a geometric mean of the two quantities a^4 and b^{16} ?

(a) $a^4 b^{16}$ (b) $a^2 b^8$ (c) $a^2 b^4$ (d) $a b^4$

(49) The geometric mean of the numbers : 2 , 5 , 8 , 10 and 125 equals .....

(a) $5\sqrt{10}$ (b) 30 (c) 8 (d) 10

(50) If x , y and z are three distinct positive real numbers in geometrical sequent , then .....

(a) $2y < x + z$ (b) $y^2 > xz$ (c) $y = xz$ (d) $\sqrt{y} = xz$

(51) In any geometric sequence , $T_1 \times T_7$ = ...

(a) $(T_1)^2$ (b) $(T_4)^2$ (c) $(T_7)^2$ (d) $(T_8)^2$

(52) If (T_n) is a geometric sequence whose first term = a and its common ratio = a , then its 5^{th} term = ...

(a) a^3 (b) a^4 (c) a^5 (d) a^6

(53) In a geometric sequence of positive terms , if the 3^{rd} term equals the multiplicative invere of the first term , then the second term = ...

(a) zero (b) – 1 (c) 1 (d) 2

(54) If the arithmetic mean of a and b equals 9 , the arithmetic mean of $\frac{1}{a}$, $\frac{1}{b}$ equals $\frac{1}{4}$, then the positive geometric mean of a , b equals .....

(a) 2 (b) 3 (c) 6 (d) 6.5

(55) If $(x - 1 , x + 2 , 3x)$ is a geometric sequence , then x = ...

(a) $\frac{1}{2}$ or – 4 (b) $-\frac{1}{2}$ or 4 (c) $\frac{1}{2}$ or 4 (d) $-\frac{1}{2}$ or – 4

(56) If $(54 , x , y , 2)$ is a geometric sequence , then $\frac{y}{x}$ = ...

(a) $\frac{1}{3}$ (b) 3 (c) 9 (d) 27

(57) If (a , b , c) form an arithmetic sequence and also form a geometric sequence , then .....

(a) $a = b \neq c$ (b) $a \neq b = c$ (c) $a \neq b \neq c$ (d) $a = b = c$

(58) If $\left(\frac{a}{b} , \frac{b}{c} , \frac{c}{d}\right)$ are three consecutive terms in a geometric sequence , then $\frac{a}{d}$ = ...

(a) $\frac{b}{c}$ (b) $\frac{b^2}{c^2}$ (c) $\frac{b^3}{c^3}$ (d) $\frac{b^4}{c^4}$

(59) If $\left(7 , m_1 , m_2 , m_3 , m_4 , \frac{7}{32}\right)$ is a geometric sequence , then m_2 = ...

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{4}$ (d) $\frac{7}{3}$

(60) If a , b are two positive real numbers and we insert three positive geometric means between a and b to form a geometric sequence with common ratio (r_1) and we insert seven positive geometric means between a and b to form another geometric sequence with common ratio (r_2) , then .....

(a) $r_1 = \frac{7}{3} r_2$ (b) $(r_1)^7 = (r_2)^3$ (c) $r_1 = \sqrt{r_2}$ (d) $r_1 = (r_2)^2$

(61) If (a , b , c , ...) is an increasing geometric sequence and (3 a , 2 b , c , ...) is an arithmetic sequence , then the common ratio of the geometric sequence = ...

(a) 2 (b) 3 (c) 4 (d) 5

(62) Two positive numbers a and b , if the first three terms in the sequence (4 , a , b , 12) are in geometric sequent and the last three terms are in arithmetic sequent , then b – a = ...

(a) 1 (b) 2 (c) $2\frac{1}{2}$ (d) 3

(63) If T_3 , T_8 , T_{18} from a not constant arithmetic sequence form a geometric sequence , then the common ratio of the geometric sequence r = ...

(a) $\frac{3}{2}$ (b) 3 (c) 2 (d) $\frac{2}{3}$

(64) If a , b , c are three different real numbers form an arithmetic sequence , their sum = 30 and a , c , b form a geometric sequence , then the greatest number of them equals .....

(a) 60 (b) 40 (c) 10 (d) – 20

(65) If (a , b , c , ...) is a geometric sequence and (a , a + b , a + c , ...) forms an arithmetic sequence , then a : b : c = ...

(a) 1 : 3 : 9 (b) 1 : 2 : 4 (c) 1 : 2 : 3 (d) 1 : 4 : 16

(66) Three non-zeroes numbers a , b , c form an arithmetic sequence. If 1 is added to a or 2 added to c , then they become a geometric sequence , then b = ...

(a) 10 (b) 12 (c) 14 (d) 16

(67) If a , b , c form an arithmetic sequence , then 10^{ax+10} , 10^{bx+10} , 10^{cx+10} where $x \neq 0$ is .....

(a) an arithmetic sequence. (b) a geometric sequence only if $x > 0$

(c) a geometric sequence only if $x < 0$ (d) a geometric sequence for all values of x

(68) The fourth term in the geometric sequence $(x , 2x + 2 , 3x + 3 , ...)$ is .....

(a) 27 (b) $\frac{27}{2}$ (c) – 27 (d) $\frac{-27}{2}$

(69) If the fourth term and the seventh term and the tenth term in a geometric sequence are x , y , z respectively , then .....

(a) $x^2 = y^2 + z^2$ (b) $y^2 = xz$ (c) $x^2 = yz$ (d) $xyz + xy - 1 = 0$

(70) If a , b are two arithmetic means between x , y and ℓ , m are two geometric means between x , y , then $\frac{a+b}{\ell m}$ = ...

(a) $\frac{x+y}{2xy}$ (b) $\frac{2xy}{x+y}$ (c) $\frac{x+y}{xy}$ (d) $\frac{xy}{x+y}$

(71) If we insert some geometric means between 2 and 1458 , and the ratio between the sum of the first two means to the last two means is 1 : 27 , then the number of this means = ...

(a) 4 (b) 5 (c) 7 (d) 8

(72) The arithmetic mean of two numbers is (m) and their geometric mean is (n) , then the sum of thier squares = ...

(a) $4m^2 - 2n^2$ (b) $m^2 + n^2$ (c) $2m^2 - n^2$ (d) $4m^2$

Second Essay questions

Exercises on the definition of the geometric sequence and its general term and identifying the geometric sequence

1 Which of the following sequences is a geometric sequence and find its common ratio , then write the first three of its terms :

(1) 📖 $(T_n) = (5 \times 2^n)$ | **(2)** 📖 $(T_n) = (4n^2)$

(3) 📖 (T_n) where $T_1 = 12$, $T_n = \frac{1}{4} T_{n-1}$, $n > 1$

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2 📖 Prove that the sequence (T_n) where $T_n = 2 \times 3^{n-5}$ is a geometric sequence , then find its seventh term. « 18 »

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3 📖 Show that the sequence (T_n) where $T_n = \frac{3}{8}(2)^n$ is a geometric sequence , then find its eighth term and the order of the term whose value is 768 « 96 , 11 »

---

4 📖 Find the next four terms of each of the following geometric sequences , then represent the first seven terms graphically :

(1) (8 , 4 , 2 ...) **(2)** $\left(\frac{1}{27} , \frac{1}{9} , \frac{1}{3} ...\right)$

---

5 In each of the following , find :

(1) A geometric sequence such that the sum of its first and second terms is 3 and the sum of its first and fourth terms is 63 « $\left(\frac{1}{2} , \frac{5}{2} , \frac{25}{2} , ...\right)$ or $(-1 , 4 , -16 , ...)$ »

(2) A geometric sequence in which the sum of its first and third terms equals 20 and its second term is 8 « (16 , 8 , 4 , ...) or (4 , 8 , 16 , ...) »

(3) 📖 A geometric sequence whose all terms are positive and its first term is four times the third term and the sum of the second and fifth terms = 36 « (64 , 32 , 16 , ...) »

(4) A geometric sequence in which its third term equals the multiplicative inverse of first term and the fifth term equals $\frac{1}{125}$, prove that there are two solutions « $\left(5 , 1 , \frac{1}{5} , ...\right)$ or $\left(5 , -1 , \frac{1}{5} , ...\right)$ »

(5) A geometric sequence in which its terms are all positive and $T_1 + T_3 = 20$, $T_3 + T_5 = 5\,T_2^2$ « $(2 , 6 , 18 , ...)$ »

(6) An increasing geometric sequence in which the third term exceeds the sum of the first two terms by 10 and the second term decreases by 14 from the sum of the first and the third terms. « $(2 , 6 , 18 , ...)$ »

(7) A geometric sequence in which the three times of the sum of the first and the third terms equals the sum of the second and the fourth terms and its fifth term exceeds the twice of the sum of the first four terms by 2 « $(2 , 6 , 18 , ...)$ »

(8) A geometric sequence whose terms are all positive and the sum of its first five terms equals 242 and $T_4 = T_3 + 6\,T_2$ « $(2 , 6 , 18 , ...)$ »

(9) A geometric sequence in which $T_2 + T_4 = 5$, $T_1^2 + T_2^2 = 80$ « $(8 , 4 , 2 , ...)$ or $(-8 , 4 , -2 , ...)$ »

(10) 🕮 If the sum of the second and third terms of a geometric sequence is 12 and the product of its first and fourth terms equals 27 « $(1 , 3 , 9 , ...)$ or $(27 , 9 , 3 , ...)$ »

6 The sum of three consecutive numbers of a geometric sequence equals 21 and their product equals 64 Find these numbers. « $1 , 4 , 16$ »

7 The sum of three consecutive positive numbers of a geometric sequence equals 14 and the product of their squares equals 4096 Find these numbers. « $8 , 4 , 2$ »

8 The sum of the first three positive terms of a geometric sequence equals 28 and the sum of their multiplicative inverses is $\frac{7}{16}$ Find these numbers. « $16 , 8 , 4$ »

9 The sum of the first three terms of a geometric sequence is 7 and the sum of their squares is 21 Find these numbers. « $4 , 2 , 1$ »

10 A geometric sequence whose common ratio is 5 and $T_{n+2} = 125$ and $T_{2n-6} = 25$ Find these geometric sequence. « $(5^{-5} , 5^{-4} , 5^{-3} , ...)$ »

11 Find the geometric sequence in which $T_n = 1$, $T_{n+1} = \frac{1}{2}$ and $T_{2n-1} = \frac{1}{32}$ Find also n « $(32 , 16 , 8 , ...) , 6$ »

12 🕮 **Discover the error :**

(1) The terms of the geometric sequence are represented by a set of separated points which are collinear.

(2) The sequence (T_n) is called geometric if $\frac{T_n}{T_{n+1}}$ equals a constant known as the common ratio of the sequence (for each $n \geq 1$)

(3) The geometric sequence is decreasing if its common ratio $r \in]-1 , 0[$

Exercises on the geometric means

13 Find the two positive numbers for which their difference is 60 and their geometric mean is 16 « 64 , 4 »

14 🕮 Find the two numbers whose arithmetic mean is 5 and their geometric mean is 3 « 9 , 1 »

15 🕮 Find the two positive numbers whose positive geometric mean is greater than one of those two numbers by 2 and is less than the other number by 3 « 4 , 9 »

16 The arithmetic mean of two numbers equals $\frac{5}{3}$ their geometric mean. Let the least number of the two numbers be 9 , so find the other number. « 81 »

17 Find the two numbers whose geometric mean exceeds their smallest one by 6 and their arithmetic mean is less than the greatest number by 9 « 6 , 24 »

18 🕮 Insert six geometric means between $\frac{1}{4}$ and 32 « $\frac{1}{2}$, 1 , 2 , 4 , 8 , 16 »

19 🕮 Find the geometric means of the sequence (4 , ... , ... , ... , ... , ... , 2916)

20 If the geometric mean between $x + 2$ and $y - 6$ is 5 and the arithmetic mean between x and y is 7 , then find x and y « 3 , 11 »

21 If $\frac{1}{2}$ a , b and c are positive quantities in a geometric sequence , prove that $4b < a + 2c$

22 🕮 If x , y , z and ℓ are positive quantities of a geometric sequence , prove that $x + \ell > y + z$

23 If we insert some positive geometric means between 2 and 486 such that the sum of the last two means equals nine times the sum of the first two means , so find the number of these means. « 4 »

24 Insert a finite geometric means between 3 and 384 if the product of the second mean and the last one is 2304 , then find the number of these means. « 6 »

25 Let (1 , x , y) be in an arithmetic sequence and (1 , y , x) be in a geometric sequence , so find the value of each of x and y such that $x \neq 1$ and $y \neq 1$ « $\frac{1}{4}$, $-\frac{1}{2}$ »

26 If 4 , b and c are arithmetically sequent and 2 , b + 3 and 5c are geometrically sequent , find the value of each of b and c « 7 , 10 »

27 The sum of three consecutive terms of an arithmetic sequence is 15 , if we subtract one from the first and one from the second and add one to the third , then they become in a geometric sequence. Find these three numbers. « 9 , 5 , 1 or 3 , 5 , 7 »

28 The sum of three consecutive numbers in a geometric sequence is 70 , if we multiply the first by 4 , the second by 5 and the third by 4 , then they become in an arithmetic sequence. Find the three numbers. « 40 , 20 , 10 »

29 Three positive numbers are in a geometric sequence their product is 8 , if we subtract one from the greatest number , then they become in an arithmetic sequence. Find the three numbers « 1 , 2 , 4 »

30 If a , 2 b , 3 c and 4 d are in a geometric sequence with positive terms , then prove that : $(a + 3c)(b + 2d) > 12bc$

31 If (AB , BC , CD , DE) are in arithmetic sequence with positive terms , then prove that : $C^2 > AE$

32 If log A , log B and log C are in an arithmetic sequence , then prove that : A , B and C are in a geometric sequence.

33 If a , 3 b , 2 c and 6 d are different positive real numbers in an arithmetic sequence , then prove that :

(1) $bc > ad$ (2) $9b^2 + 4c^2 > 2ac + 18bd$

34 🕮 **Discover the error :**

(1) The geometric means are known that they are the terms lying between two non-consecutive terms of a geometric sequence and can be found whenever the values of those two terms are known.

(2) The arithmetic mean of two different real numbers is greater than thier geometric mean.

35 **Creative thinking :**

(1) 🕮 If $a + b + c = 1$ where a , b , c are different and positive quantities , **prove that :** $(1 - a)(1 - b)(1 - c) > 8abc$

(2) 🕮 If $x \in \mathbb{R}^+$, $x \neq 1$, prove that : $x + \frac{1}{x} > 2$

(3) If $abcd = 1$ where a , b , c , d are positive different quantities , **prove that :** $(a + 1)(b + 1)(c + 1)(d + 1) > 16$

Third Higher skills

Choose the correct answer from those given :

(1) If (T_n) , $(\acute{T_n})$ are two geometric sequences , which of the following is a geometric sequence ?

(a) (T_n^k) (b) (kT_n) (c) $(T_n \acute{T_n})$ (d) All the previous.

(2) If (a , b , c) is an arithmetic sequence its common difference (m) , then $(3^a , 3^b , 3^c)$ is .....

(a) an arithmetic sequence , its common difference = 3

(b) a geometric sequence , its common ratio = 3

(c) an arithmetic sequence , its common difference = 3^m

(d) a geometric sequence , its common ratio = 3^m

(3) If (a , b , c) form an arithmetic sequence and (x , y , z) form a geometric sequence then $x^{b-c} . y^{c-a} . z^{a-b}$ = ...

(a) x y z (b) 1 (c) x + y + z (d) a + b + c

(4) If the arithmetic mean and the geometric mean of the roots of a quadratic equation are 8 , 5 respectively , then the equation is .....

(a) $x^2 - 16x - 25 = 0$ (b) $x^2 - 8x + 5 = 0$

(c) $x^2 - 16x + 25 = 0$ (d) $x^2 + 16x - 25 = 0$

(5) If the altitudes of triangle ABC drawn from its vertices A , B and C respectively form an arithmetic squence , then .....

(a) a , b , c form an arithmetic sequence. (b) a , b , c form a geometric sequence.

(c) ab , ac , bc form an arithmetic sequence. (d) ab , bc , ca form a geometric sequence.

(6) If (a , b , c , ...) is a geometric sequence and $a^x = b^y = c^z$, then .....

(a) $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$ (b) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$ (c) $\frac{1}{y} + \frac{1}{z} = \frac{2}{x}$ (d) $\frac{x}{z} + 1 = \frac{2}{y}$

Life applications

Applications on the geometric sequence

1 **Percentage :** The price of a new brand car is L.E. 150000 How much is that car after 4 years if its price decreases at a rate of 10 % yearly ?

« L.E. 98415 »

2 **Income :** The salary of an emplyee is L.E. 1200 and he gets a yearly constant raise at a rate of 10 % more than his salary in the directly previous year. How much is his salary after 4 years ?

« L.E. 1756.92 »

3 **Environment :** Water is poured in a water tank at a rate of twice the directly previous day. How many days does the water tank hold 1536 liters if 12 liters has been poured in the tank on the first day ?

« 8 days »

4 **Education :** If the number of the students accepted in the high school stage in an educational administration increases at a rate of 4 % yearly. How many students will be there after 6 years if the number of the students is 2400 students right now ?

« 3037 students »

5 **Physics :** A rubber ball falls from a height of 240 metres above the ground surface. How high is the ball after the seventh bounce , If the ball bounces back $\frac{3}{4}$ of its directly previous height ?

« 32 metres »

Exercise

6

Geometric series

From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

Exercises on the geometric series and the sum of the first n terms of a geometric series :

(1) The sum of the geometric series in which $a = \frac{1}{2}$, $r = -2$, $n = 10$ equals .....

(a) – 170.5 (b) 158 (c) 164 (d) – 164

(2) If a geometric sequence whose first term $a = 2$ and its common ratio $r = 1$, then the sum of the first 10 terms of it = ...

(a) 20 (b) 2 (c) 10 (d) 1024

(3) The sum of the geometric series in which $a = 9$, $r = 3$, $\ell = 6561$ is .....

(a) 29524 (b) 9837 (c) 2954 (d) 8937

(4) The sum of the first 8 terms of the geometric series $\frac{1}{4} + \frac{1}{2} + 1 + 2 + ...$ is .....

(a) $63\frac{3}{4}$ (b) 32 (c) $31\frac{3}{4}$ (d) 64

(5) The sum of 9 terms in the geometric sequence $\left(1 , -\frac{1}{2} , \frac{1}{4} , ...\right)$ equals .....

(a) $\frac{171}{256}$ (b) $\frac{-85}{128}$ (c) $\frac{-85}{256}$ (d) $\frac{178}{256}$

(6) $3 + 6 + 12 + ... + 192 =$ .....

(a) 192 (b) 381 (c) 189 (d) 765

(7) The sum of the geometric sequence $(3 , -6 , 12 , ... , 768)$ equals .....

(a) – 98 (b) – 314 (c) 498 (d) 513

(8) The sum of the first eight terms of the geometric sequence (T_n) where $T_n = 2\,(3)^{n+1}$ equals .....

(a) 56014 (b) 58094 (c) 59040 (d) 4950

(9) The sum of five terms of the geometric sequence (1 , 3 , 9 , ...) starting from its third term equals .....

(a) 1089 (b) 2013 (c) 998 (d) 1054

(10) If a geometric series whose first term a = 2 and its common ratio $r = \frac{1}{2}$, then $\sum_{i=1}^{6} ar^{i-1}$ = ...

(a) $3\frac{15}{16}$ (b) $\frac{1}{16}$ (c) $3\frac{7}{8}$ (d) $\frac{1}{32}$

(11) $\sum_{i=3}^{7} (2 \times 3^{i-1})$ = ...

(a) 242 (b) 1458 (c) 738 (d) 2178

(12) A sequence in which the sum of the first n terms is given by the relation $S_n = 3^{n+1} - 4$, then the third term equals .....

(a) 18 (b) 23 (c) 54 (d) 77

(13) The number of terms must be taken from the geometric sequence (3 , 6 , 12 , ...) starting from its first term to make the sum of these terms = 381 is .....

(a) 8 (b) 6 (c) 9 (d) 7

(14) The number of terms must be taken from the geometric sequence (2 , 6 , 18 , ...) starting from the second term such that its sum is 6558 is .....

(a) 7 (b) 8 (c) 9 (d) 10

(15) The geometric sequence whose first term = 243 , last term = 1 , and the sum of its terms = 364 is .....

(a) (243 , 27 , 3 , ...) (b) (729 , 243 , ... , 1)

(c) (243 , 81 , ... , 1) (d) (243 , 121.5 , 60.75 , ... , 1)

(16) The geometric sequence whose sum is 1093 , last term is 729 and its common ratio is 3 is .....

(a) (1 , 3 , 9 , ... , 729) (b) (2 , 6 , 18 , ... , 729)

(c) (3 , 9 , 27 , ... , 729) (d) (– 3 , 9 , – 27 , ... , 729)

(17) The least number of terms that must be taken from the geometric sequence (5 , 15 , 45 , ...) starting from its first term such that their sum is greater than 6400 is ..... terms.

(a) 5 (b) 6 (c) 7 (d) 8

(18) The sum of the first five terms in a geometric sequence = 7.75 and the sum of the next 5 terms = 248 , then the sequence is .....

(a) (4 , 2 , 1 , ...) (b) $\left(\frac{1}{4}, \frac{1}{2}, 1, ...\right)$

(c) (2 , 4 , 8 , ...) (d) $\left(\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, ...\right)$

(19) A geometric sequence , the sum of its fourth term and sixth term = 120 and the sum of its fifth term and seventh term = 240 , then the sum of the first 10 terms from the sequence = ...

(a) 720 (b) 1023 (c) 3069 (d) 6138

(20) The terms of a geometric sequence are positive and different , the sum of the first twelve terms equals 273 times the sum of its first four terms , then the common ratio = ...

(a) ± 4 (b) ± 2 (c) 17 (d) 2

(21) The number of terms of a geometric sequence is (2n) and its common ratio is (r) , then ratio between the sum of its odd-ordered terms to the even-ordered terms equals .....

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) r^2 (d) $\frac{n}{r}$

(22) The number of terms in a geometric sequence is (2n) and the sum of all terms equals five times the sum of its odd ordered terms , then the common ratio of the sequence = ...

(a) 2 (b) 3 (c) 4 (d) 5

Exercises on the infinite geometric series - sum of the infinite geometric sequence

(23) We can find the sum of an infinite number of terms of a geometric sequence if and only if .....

(a) $r < 1$ (b) $r > 1$ (c) $|r| < 1$ (d) $|r| > 1$

(24) The sum of the terms of the geometric sequence (81 , 27 , 9 , ...) is .....

(a) $\frac{243}{4}$ (b) 117 (c) 118 (d) $\frac{243}{2}$

(25) The sum of an infinite number of terms of the geometric sequence (25 , – 5 , 1 , ...) equals .....

(a) 22 (b) 21 (c) $20\frac{5}{6}$ (d) $21\frac{3}{4}$

(26) The sum of an infinite number of terms of the geometric sequence $\left(3, \sqrt{3}, 1, ...\right)$ equals .....

(a) $\frac{5+2\sqrt{3}}{2}$ (b) $\frac{9+3\sqrt{3}}{2}$ (c) $\frac{8+4\sqrt{3}}{3}$ (d) $3+\sqrt{3}$

(27) The sum of an infinite number of terms of the geometric sequence $(3^{-1}, (-3)^{-3}, (3)^{-5}, ...)$ equals .....

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 0.3333 (d) 0.3

(28) The sum of infinite number of terms of the geometric sequence $(T_n) = (3)^{3-n}$ equals .....

(a) 13 (b) $13\frac{1}{2}$ (c) $13\frac{1}{3}$ (d) $12\frac{1}{2}$

(29) $\sum_{i=1}^{\infty} 4\left(\frac{1}{2}\right)^{i-1}$ = ...

(a) 8 (b) 4 (c) $4\frac{1}{2}$ (d) 2

(30) $\sum_{n=1}^{\infty} 24\left(\frac{-1}{5}\right)^{n-1}$ = ...

(a) 50 (b) 40 (c) 20 (d) 10

(31) $\sum_{r=1}^{\infty} 27\,(3)^{1-r}$ = ...

(a) 40 (b) $\frac{81}{5}$ (c) 39 (d) $\frac{81}{2}$

(32) The geometric series $48 + 24 + 12 + ...$ by using the summation notation is .....

(a) $\sum_{r=1}^{\infty} 48 \times 2^{r-1}$ (b) $\sum_{r=1}^{\infty} (24)^{r-1}$

(c) $\sum_{r=1}^{\infty} 48\,(2)^{1-r}$ (d) $\sum_{r=1}^{\infty} 24 \times \left(\frac{1}{2}\right)^{r-1}$

(33) $0.\overline{24}$ = ...

(a) $\frac{6}{25}$ (b) $\frac{2}{11}$ (c) $\frac{11}{45}$ (d) $\frac{8}{33}$

(34) $0.\overline{57}$ = ...

(a) $\frac{57}{100}$ (b) $\frac{25}{33}$ (c) $\frac{23}{40}$ (d) $\frac{19}{33}$

(35) $0.\overline{432}$ = ...

(a) $\frac{214}{495}$ (b) $\frac{389}{900}$ (c) $\frac{43}{99}$ (d) $\frac{16}{37}$

(36) If the sum of an infinite number of terms of a geometric sequence whose common ratio is $\frac{1}{3}$ is $13\frac{1}{2}$, then its first term equals .....

(a) 6 (b) 8 (c) 9 (d) 12

(37) If the sum of an infinite number of terms of a geometric sequence whose first term is 12 is 96, then its common ratio equals .....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

(38) The sum of an infinite number of terms of the geometric sequence (T_n) in which $T_1 = 1$, $T_n = 2\,T_{n+1}$ is .....

(a) ∞ (b) 2 (c) 3 (d) 4

(39) The sum of squares of the terms of an infinite geometric sequence whose first term = 1 and common ratio = y is .....

(a) $\frac{1}{1-y}$ (b) $\frac{1}{1-y^2}$ (c) $\frac{1}{y^2}$ (d) $\frac{1}{y}$

(40) The sum of the series $\left(1+\frac{1}{x}+\frac{1}{x^2}+...\right)$ equals ..... (where $|x|>1$)

(a) $\frac{1}{1-x}$ (b) $\frac{x}{1-x}$ (c) $\frac{x}{x-1}$ (d) $\frac{x}{x^2-1}$

(41) A geometric sequence whose first term equals the sum of the next infinite terms , then its common ratio equals .....

(a) 0.5 (b) 0.333 (c) 0.25 (d) 0.666

(42) If the first term in an infinite geometric sequence equals twice the sum of all its next terms , then the common ratio of the sequence = ...

(a) 1 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

(43) If $(96 , x , y , z , 6 , ...)$ is a geometric sequence , all its terms are positive , then the sum of an infinite terms of its terms = ...

(a) 180 (b) 192 (c) 384 (d) 768

(44) In a geometric sequence , $T_2 = 240$, $T_5 = 30$, then the sum of infinite terms of the sequence = ...

(a) 1920 (b) 960 (c) 480 (d) 240

(45) The sum of an infinite number of terms of a geometric sequence is 54 and its first term is 18 , then the is sequence is .....

(a) $(18 , 6 , 2 , ...)$ (b) $\left(18 , 9 , \frac{9}{2} , ...\right)$

(c) $\left(18 , 13.5 , \frac{81}{8} , ...\right)$ (d) $(18 , 12 , 8 , ...)$

(46) The infinite geometric sequence whose terms are positive , its first term is greater than its second term by 30 and the sum of an infinite number of its terms equals $\frac{135}{2}$ is .....

(a) $(45 , 15 , 5 , ...)$ (b) $(60 , 30 , 15 , ...)$

(c) $(90 , 60 , 20 , ...)$ (d) $\left(32 , 2 , \frac{2}{16} , ...\right)$

(47) A geometric sequence whose sum to infinity is 4. If the sum of the cubes of its terms to infinity is 192 , then geometric sequence is .....

(a) $\left(6 , 3 , \frac{3}{2} , ...\right)$ (b) $\left(8 , 6 , 4\,\frac{1}{2} , ...\right)$

(c) $\left(6 , -2 , -\frac{2}{3} , ...\right)$ (d) $\left(6 , -3 , \frac{3}{2} , ...\right)$

(48) Each term in a geometric sequence equals half the sum of the next terms to infinity if the sum of its second term and fourth term is $8\frac{2}{3}$, then the sequence is .....

(a) $\left(\frac{27}{8} , \frac{9}{4} , \frac{3}{2} , ...\right)$ (b) $\left(1 , \frac{3}{2} , \frac{9}{4} , ...\right)$

(c) $(9 , 6 , 4 , ...)$ (d) $\left(1 , \frac{2}{3} , \frac{4}{9} , ...\right)$

(49) The terms of a geometric sequence are positive , the sum of its second and third terms equals 20 , and the sum of the first three terms equals 65 , then the sum of its terms to infinity equals .....

(a) 42.5 (b) 67.5 (c) 78.5 (d) 170

(50) If the sum of the first two terms from an infinite geometric sequence equals 1 and each term is equal to twice the sum of all its next terms , then its first term = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

(51) The value of the series $\left(1 + \frac{1}{3} + \frac{1}{3^2} + ...\right)$ is .....

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{4}{3}$ (d) $\frac{10}{9}$

(52) If the sum of infinite terms of a geometric sequence is 3 times the sum of its first two terms , then the common ratio = ...

(a) $\pm\frac{\sqrt{2}}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\pm\sqrt{\frac{2}{3}}$ (d) $\frac{\sqrt{2}}{3}$

(53) If $S = 1 + a + a^2 + ... \infty$ where $|a| < 1$

, $\grave{S} = 1 + b + b^2 + ... \infty$ where $|b| < 1$, then $\frac{S}{\grave{S}}$ = ...

(a) $\frac{a}{b}$ (b) $\frac{a+1}{b+1}$ (c) $\frac{1-a}{1-b}$ (d) $\frac{1-b}{1-a}$

(54) Infinite geometric sequence whose second term equals 2 and sum of infinite number of its terms equals 8 , then its common ratio = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(55) The value of the series $(1 + \cos^2\theta + \cos^4\theta + ...)$ equals ..... where $\theta \neq n\pi$, $n \in \mathbb{Z}$

(a) $\sin^2\theta$ (b) $\csc^2\theta$ (c) $\cos^2\theta$ (d) $\sec^2\theta$

(56) If $1 + \cos\theta + \cos^2\theta + \cos^3\theta + ... = 2 - \sqrt{2}$

, then θ = ... where $\theta \in]0 , \pi[$

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{3\pi}{8}$ (d) $\frac{3\pi}{4}$

(57) The product of $9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \times ...$ to ∞ equals .....

(a) 3 (b) 6 (c) 9 (d) 1

(58) If S_n is the sum of n terms from the infinte sequence $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ and $S_\infty - S_n < \frac{1}{1000}$, then the least value of n = ...

(a) 8 (b) 9 (c) 10 (d) 11

Second Essay questions

Exercises on the infinite geometric series - sum of the infinite geometric sequence

1 **Find the sum of each of the following geometric series :**

(1) 1 + 3 + 9 + ··· + 6561 (2) $20 - 10 + 5 - \cdots - \frac{5}{32}$

2 **Find the sum of each of the geometric sequences in which :**

(1) a = 4 , r = 3 , n = 6 (2) $a = -4 , r = -\frac{1}{2} , \ell = \frac{1}{8}$

3 **Find :**

(1) $\sum_{i=1}^{5} 2 \times \left(\frac{1}{2}\right)^{i-1}$ (2) $\sum_{i=5}^{12} 3 \times 2^{i-1}$

4 Prove that the sequence $(T_n) = (10 \times 2^{n-2})$ is a geometric sequence , then find the number of the terms must be taken from the sequence starting from the first term such that their sum is 2555 « 9 »

5 A geometric sequence whose first term is 2 and its fourth term is 54 , find the least number of terms of this geometric sequence must be taken starting from its first term such that their sum is greater than 5000 « 8 »

6 A geometric sequence , if the fourth term equals 8 and the seventh term equals 64 , find this sequence and the sum of its first ten terms. « (1 , 2 , 4 , ···) , 1023 »

7 (T_n) is a geometric sequence whose terms are positive in which $T_2 = 6$ and $T_3 - T_1 = 9$ Find the sequence and the sum of the first twelve terms. « (3 , 6 , 12 , ···) , 12285 »

8 A geometric sequence all its terms are positive , the first term equals four times of the third one and the sum of the second and fifth terms equals 36 , then find this geometric sequence and the sum of the first ten terms. « (64 , 32 , 16 , ···) , $127\frac{7}{8}$ »

9 A geometric sequence , all its terms are positive. If $T_3 + T_4 = 6\,T_2$ and $T_7 = 320$, find this geometric sequence and the sum of its first eight terms. « (5 , 10 , 20 , ···) , 1275 »

10 A geometric sequence whose sum of its first three terms equals 13 and the sum of the directly next three terms is 351 , find the sequence and the sum of its first ten terms. « (1 , 3 , 9 , ···) , 29524 »

11 The sum of the first four terms of a geometric sequence is 60 and the sum of the next four terms is 16 times the sum of the first four terms. Find this sequence.

« (4 , 8 , 16 , ···) or (–12 , 24 , – 48 , ···) »

12 Let the sum of the first n terms from a sequence be given by : $S_n = \frac{1}{2}[3^{n+2} - 9]$
Prove that this sequence is a geometric sequence , then find it. « (9 , 27 , 81 , ···) »

13 As inserting n geometric means between 81 and $\frac{1}{729}$, the sum of the first two means is 36 and the sum of the last two means is $\frac{4}{243}$. Find the sum of these geometric means.

« $\frac{9841}{243}$ »

14 If the sum of the first nine terms of a geometric sequence is ℓ , and the sum of the next nine terms is m , prove that the sequence common ratio = $\sqrt[9]{\frac{m}{\ell}}$

Exercises on the infinite geometric series - sum of the infinite geometric sequence

15 **Which of the following geometric series can be summed up to infinity , then find its sum (if possible) :**

(1) 7 + 21 + 63 + ···
(2) 75 + 45 + 27 + ···
(3) $15 - 5 + \frac{5}{3} - \frac{5}{9} + \cdots$
(4) 96 – 48 + 24 – 12 + ···

16 **Which of the following geometric sequences can be summed up to infinity , then find its sum (if possible) :**

(1) (24 , 12 , 6 , ···)
(2) (3 , – 6 , 12 , ···)
(3) $(T_n) = (5 \times 2^n)$
(4) $(T_n) = (2 \times 5^{1-n})$

17 **Change each of the following circulating decimals into a common fraction :**

(1) $0.\overline{3}$ « $\frac{1}{3}$ »
(2) $0.\overline{18}$ « $\frac{2}{11}$ »
(3) $\sqrt[3]{0.\overline{037}}$ « $\frac{1}{3}$ »
(4) $0.\overline{4} + 0.2\overline{3}$ « $\frac{61}{90}$ »

18 **Find :**

(1) $\sum_{i=1}^{\infty} 56\left(\frac{3}{4}\right)^{i-1}$ « 224 »
(2) $\sum_{i=2}^{\infty} \frac{1}{81}(3)^{1-i}$ « $\frac{1}{162}$ »

19 The first term of a geometric sequence of an infinite number of terms = 18 and the fourth term = $\frac{16}{3}$ What is the sum of this sequence ? « 54 »

20 A geometric sequence in which the sum of an infinite number of its terms starting from its first term equals 108 and its first term is greater than its second term by 12. Find the sequence and the sum of its first seven terms. « (36 , 24 , 16 , ···) , $\frac{8236}{81}$ »

21 Find the geometric sequence whose sum of its first and second terms = 16 , and the sum of an infinite number of its terms = 25 « $(10 , 6 , \frac{18}{5} , \cdots)$ or $(40 , -24 , \frac{72}{5} , \cdots)$ »

22 Find the infinite geometric sequence whose first term = the sum of the next infinite terms of it and the sum of its first and second terms = 9 « $(6 , 3 , \frac{3}{2} , \cdots)$ »

23 A geometric sequence , its terms are positive and each of its terms equals twice the sum of the following terms to ∞ , if its third term is equal to the multiplicative inverse of its fifth term , then find this geometric sequence and the sum of its first five terms.
« $(27 , 9 , 3 , \cdots) , \frac{121}{3}$ »

24 Find the geometric sequence in which each of its terms equals 7 times the sum of the following terms to ∞ and its third term is $\frac{3}{8}$ « $(24 , 3 , \frac{3}{8} , \cdots)$ »

25 If the sum of an infinite geometric sequence is $\frac{375}{4}$, and the sum of its first and second terms is 90 , prove that there are two sequences and find them.
« $(75 , 15 , 3 , \cdots)$ or $(112.5 , -22.5 , 4.5 , \cdots)$ »

26 (T_n) is a geometric sequence in which $T_2 - T_6 = 45$ and the first $S_4 = 180$ Find the sequence and show that there are an infinite number of its terms can be summed and find this sum. « $(96 , 48 , 24 , \cdots) , 192$ »

27 The terms of a geometric sequence are positive and the sum of the first and the second terms is 108 and the sum of the third and the fourth terms is 12 , find the sequence and show that it is possible to sum up it to infinity and find this sum. « $(81 , 27 , 9 , \cdots) , 121\frac{1}{2}$ »

28 (T_n) is a geometric sequence in which $T_1 + T_4 = 70$, $T_2 + T_3 = 60$, prove that there are two sequences and an infinite number of the terms of one of them can be summed and find this sum starting from its first term. « 162 »

29 A geometric sequence in which the product of its first three terms is 64 and the sum of the second , the third and the fourth terms is 7 , prove that there are two geometric sequences such that we can sum the terms of one of them to ∞ and find this sum. « 16 »

30 The sum of terms of a geometric sequence to ∞ is 18 and the sum of the squares of its terms to ∞ is 108 , find this geometric sequence. « $(9 , \frac{9}{2} , \frac{9}{4} , \cdots)$ »

31 If the sum of the first three terms of a geometric sequence is 14 and the sum of their squares is 84 prove that there are two sequences and one of them can be summed to ∞ , then find this sum.

32 **Discover the error :**

(1) The sum of an infinite geometric series can be found when $|r| \leq 1$

(2) The sum of an infinite number of terms of the sequence (16 , 8 , 4 , ...) is greater than twice its first term.

Third Higher skills

Choose the correct answer from those given :

(1) The first two terms of an infinite geometric sequance are positive integers , their sum = 3 , then S_∞ = ...

(a) 4 (b) 8 (c) 64 (d) 1024

(2) If $\sum_{r=0}^{\infty} \frac{a}{(4)^r} = 16$, then a = ...

(a) 4 (b) 8 (c) 12 (d) 16

(3) The first term of an infinite geometric sequence = $\mathcal{X}$ and the sum of its terms = 5 , then .....

(a) $\mathcal{X} \geq 10$ (b) $0 < \mathcal{X} < 10$ (c) $\mathcal{X} < -10$ (d) $-10 < \mathcal{X} < 0$

(4) In a geometric sequence , S_{18} (the first 18 terms) – S_{10} (the first 10 terms) = $\mathcal{X}$ and S_{15} (the first 15 terms) – S_7 (the first 7 terms) = y , then r = ...

(a) $\frac{\mathcal{X}}{y}$ (b) $\sqrt{\frac{\mathcal{X}}{y}}$ (c) $\sqrt[3]{\frac{\mathcal{X}}{y}}$ (d) $\sqrt{\frac{y}{\mathcal{X}}}$

(5) The first term in a geometric sequence is (a) and its common ratio is (r) and the number of its terms is (n) , the sum of its terms is (S) , then the sum of the reciprocals of these terms is .....

(a) $\frac{S}{ar^{n-1}}$ (b) $\frac{aS}{r^{n-1}}$ (c) $\frac{2S}{ar^{n-1}}$ (d) $\frac{S}{a^2 r^{n-1}}$

(6) The first term of a geometric sequence is (a) and the last term is (ℓ) , the number of its terms is (n) , then the product of all terms is .....

(a) $\left(\frac{a}{\ell}\right)^{\frac{n}{2}}$ (b) $(a\ell)^n$ (c) $a(\ell)^{\frac{n}{2}}$ (d) $(a\ell)^{\frac{n}{2}}$

(7) If $\mathcal{X} = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\theta$, then .....

(a) $\mathcal{X} y = 1$ (b) $\mathcal{X} y = \mathcal{X} + y$ (c) $\mathcal{X} + y = 1$ (d) $\mathcal{X} = y$

(8) If T_n is the n^{th} term in a geometric sequence , its terms are positive , and $a = \sum_{n=1}^{100} T_{2n}$, $b = \sum_{n=1}^{100} T_{2n+1}$ where $a \neq b$, then the common ratio of the geometric sequence = ...

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

(9) The sum of the series $(1 + 2\mathcal{X} + 3\mathcal{X}^2 + 4\mathcal{X}^3 + \ldots \text{ to } \infty)$ for all $0 < \mathcal{X} < 1$ equals .....

(a) $\frac{1}{1-\mathcal{X}}$ (b) $\frac{1}{1+\mathcal{X}}$ (c) $\frac{1}{1-2\mathcal{X}}$ (d) $\frac{1}{(1-\mathcal{X})^2}$

(10) If L , M are the roots of the equation $16X^2 + 1 = 16X$, then $\sum_{r=1}^{\infty} L^r + \sum_{r=1}^{\infty} M^r =$ .....

(a) 12 (b) 14 (c) 16 (d) 32

(11) The sum of the first 20 terms in the sequence (1 , 2 , 3 , 4 , 5 , 8 , 7 , 16 , ...) is .....

(a) 512 (b) 1023 (c) 1123 (d) 2146

Life applications

1 **Biology :** Culturing bacteria is duplicated every day (in a nutritional medium). How many bacteria are there after ten days if their number is 800 on the first day ? « 818400 »

2 A container is full of 6138 litres of water. If the volume of the water decreased in three consecutive days by 6 , 12 , and 24 litres respectively , after how many days will the container be empty , given that the decrease is by the same rate ? « 10 days »

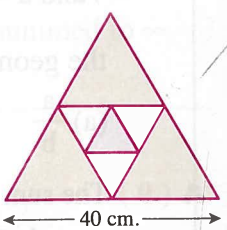
3 **Income :** A worker has started to work at a factory for a yearly income of L.E. 7200 so that he got a yearly raise of 6 % of the directly previous year. Calculate his income in the seventh year and the sum of what he got within the first seven years.

« L.E. 10213 , L.E. 60436 »

4 An agricultural crops storing company has seven warehouses to store the wheat so that the first warehouse holds 270 tons and each warehouse after that can hold two third of the amount of the directly previous warehouse. Can the company store 800 tons of wheat ? what is the greatest amount of wheat the company can store in its warehouses to the nearest ton ? « 763 tons »

5 **Mining :** A gold mine produces 4200 kg. in the first year. If the production of that mine decreases at a rate of 10 % yearly of the production of the directly previous year , find the production of that mine in the eighth year , then calculate the production of the mine within the first eight years. « 2008.8 kg. , 23920 kg. »

6 **Geometry :** The opposite figure shows an equilateral triangle whose side length is 40 cm. Another triangle is drawn interiorly through connecting the points representing the mid-points of the greatest triangle. The interior triangles are repeatedly drawn the same way , find the sum of the perimeters of the first 10 triangles in this pattern to the nearest integer.



« 240 cm. »

7 Which of the following gives more income during 25 years : A job which starts with an annual salary of L.E. 1000 with a constant annual raise of L.E. 30 , or a job which starts with the same annual salary with an annual raise of 2 % of the salary of the preceding year ? Then find the difference between the two incomes. « L.E. 1970 »

8 The production of an oil well is decreasing at a rate of 5% compared to the last year's production. If the production of the first year is 48000 barrels , then find the maximum production of the well. « 960000 barrels »

9 **Physics :** A small ball of iron has been rolled over on a horizontal plane. If the ball covered 25 m. in the first minute , then it started to cover 60% only in each next minute from the distance it covered in the directly previous minute. Find the total distance which the ball covered till it stopped completely. « $\frac{125}{2}$ m. »

10 A rubber ball falls from a height of 10 metres on the ground and rebounces vertically to half the height that it falls from in each time it rebounces up. Find the sum of the distances which the ball covered till to be at rest. « 30 m. »

Unit Two

Permutations and combinations

Scan the QR code to solve an interactive test on each lesson

Exercise 7 Counting principle - Permutations.

Exercise 8 Combinations.

Exercise

7

Counting principle - Permutations

From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\lfloor 0 + \lfloor 1 + \lfloor 2 + \lfloor 3 =$ .....

(a) 6 (b) 8 (c) 9 (d) 10

(2) ${}^{9}P_{0} + {}^{7}P_{1} + {}^{3}P_{3} =$ .....

(a) 14 (b) 25 (c) 96 (d) 189

(3) If $\frac{1}{2}\lfloor n = 1$, then $\lfloor n - 2 =$ .....

(a) 4 (b) 2 (c) 1 (d) zero

(4) $\frac{\lfloor n}{n} =$ .....

(a) 1 (b) $\lfloor n$ (c) $\lfloor n - 1$ (d) $\frac{n}{n-1}$

(5) ${}^{n}P_{2}$ can be equal to .....

(a) 15 (b) 16 (c) 17 (d) 20

(6) If ${}^{5}P_{r} = 60$, then r = ...

(a) 4 (b) 3 (c) 2 (d) 5

(7) If ${}^{n}P_{3} = 120$, then n = ...

(a) 6 (b) 5 (c) 4 (d) 3

(8) The solution set of the equation $\lfloor x = 1$ in $\mathbb{R}$ is .....

(a) $\{1\}$ (b) $\{0\}$ (c) $\{0 , 1\}$ (d) $\{1 , -1\}$

(9) If $10 \lfloor 2n - 2 = 240$, then n = ...

(a) 4 (b) 2 (c) 3 (d) 5

(10) If $\frac{1}{\lfloor x} = \frac{2}{\lfloor x+1}$, then x = ...

(a) 0 (b) 1 (c) 2 (d) 3

(11) $\frac{{}^{7}P_{r}}{{}^{7}P_{r-1}}$ = ...

(a) r (b) r – 1 (c) 7 – r (d) 8 – r

(12) If ${}^{9}P_{r-1} = 504$, then $\lfloor r+1$ = ...

(a) 5 (b) 24 (c) 120 (d) 720

(13) If $\lfloor \frac{1}{2}n = 120$, then ${}^{n}P_{3}$ = ...

(a) 30 (b) 60 (c) 120 (d) 720

(14) If $\frac{1}{\lfloor 9} + \frac{1}{\lfloor 10} = \frac{x}{\lfloor 11}$, then x = ...

(a) 1 (b) 11 (c) 121 (d) 132

(15) $n \lfloor n + \lfloor n = \lfloor$ .....

(a) n (b) n + 1 (c) n + 2 (d) n – 1

(16) The number of ordered pairs (a , b) which can be formed from the elements of the set $\{1 , 2 , 3\}$ where $a \neq b$ is .....

(a) 2 (b) 3 (c) 6 (d) 9

(17) The number of ways to arrange 5 persons sitting on a circle is .....

(a) 1 (b) 5 (c) 24 (d) 120

(18) The number of ways to arrange 4 students sitting on 4 seats in one row equals .....

(a) 1 (b) 4 + 4 (c) 4 × 4 (d) 4 × 3 × 2 × 1

(19) The number of ways to select a meal and a drink from a list contains 5 meals and 4 drinks is .....

(a) 9 (b) 20 (c) 5 (d) 7

(20) There are four different chemistry books and three different history books and two different poetry books by how many ways a book of each subject can be chosen ?

(a) 4 + 3 + 2 (b) 4 × 3 × 2 (c) 1 + 1 + 1 (d) 1 × 1 × 1

(21) If a man chooses a car from the brands {Opel , Kia , Honda} and the available colors are {white , black , silver , red} , then the number of ways to choose the car equals .....

(a) 7 (b) 12 (c) 14 (d) 24

(22) The number of three different digit numbers can be formed from the numbers $\{1, 3, 5, 6\}$ is .....

(a) 9 (b) 12 (c) 24 (d) 64

(23) How many ways can a president and a vice president be selected from a 12 members committee ?

(a) 2 (b) 23 (c) 66 (d) 132

(24) The number of ways to arrange the letters of the word «HELP» equals .....

(a) 4 (b) 9 (c) 10 (d) 24

(25) The number of two different digit numbers can be formed from the numbers $\{5, 3, 0, 4\}$ equals .....

(a) 3×2 (b) 4×2 (c) 3×3 (d) 3×4

(26) The number of the odd three different digit numbers formed from $\{2, 3, 4, 6\}$ equals .....

(a) $8 \times 6 \times 3$ (b) $4 \times 3 \times 3$ (c) $4 \times 3 \times 2$ (d) $1 \times 3 \times 2$

(27) The number of ways to form a three different digit prime number of the set of the numbers 3 , 4 , 5 is .....

(a) 6 (b) 3 (c) 1 (d) zero

(28) The number of ways to form the number 1453 from the numbers 1 , 3 , 4 , 5 is .....

(a) 24 (b) 16 (c) 1 (d) zero

(29) The number of ways to form a 3-digit number of 6 numbers except zero is .....

(a) $6 \times 5 \times 4$ (b) $6 + 5 + 4$ (c) $6 \times 6 \times 6$ (d) $3 \times 2 \times 1$

(30) How many even numbers can be formed from 4 different digits from the set $\{1, 3, 4, 5\}$?

(a) 5431 (b) 60 (c) 12 (d) 6

(31) How many numbers can be formed from 3 different digits from the set $\{2, 4, 5, 7\}$ and be less than 500 ?

(a) 7 (b) 8 (c) 12 (d) 24

(32) The number of ways can 10 true-false questions be answered is .....

(a) $\underline{|10}$ (b) 10 (c) 2^{10} (d) 10^2

(33) The number of 5-digit numbers can be formed from 0 , 1 , 2 , 3 , 4 equals .....

(a) 2500 (b) 96 (c) 120 (d) 3125

(34) The number of ways to form a 4-digit number formed from different digits using the digits $\{2, 3, 4, 7\}$ such that the tens digits is even is .....

(a) 15 (b) 24 (c) 12 (d) 8

(35) $\frac{\lfloor r-2}{\lfloor r-3} =$ .....

(a) $r-2$ (b) $r-3$ (c) r (d) $r-1$

(36) If ${}^{n}P_{n} = 5040$, then n = ...

(a) 10 (b) 5 (c) 7 (d) 3

(37) If $\lfloor n+1 = 30 \lfloor n-1$, then n = ...

(a) 5 (b) 6 (c) 29 (d) 30

(38) If $\lfloor n-1 = 120$, $\lfloor n-r = 6$, then ${}^{n}P_{r} =$ .....

(a) 720 (b) $6 \times 5 \times 4$ (c) $3 \times 4 \times 5$ (d) 6×5

(39) If $\frac{\lfloor n-3}{\lfloor 3} = \frac{\lfloor 5}{n-2}$, then ${}^{n}P_{2} =$ .....

(a) 8 (b) 28 (c) 56 (d) 336

(40) If $n \lfloor 2n-1 = 60$, then n = ...

(a) 2 (b) 2.5 (c) 4 (d) 5

(41) If $8 \times 7 \times 6 = {}^{x}P_{y}$, then $x + y$ could be equal to .....

(a) 35 (b) 18 (c) 13 (d) 11

(42) If $\frac{\lfloor 2n \lfloor n-1}{\lfloor 2n-1 \lfloor n+1} = \frac{1}{3}$, then n = ...

(a) 3 (b) 5 (c) 7 (d) 9

(43) The solution set of the equation : $\lfloor n-1 = n$ in $\mathbb{Z}$ is .....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $\{1, 2\}$

(44) $\frac{2 \times 4 \times 6 \times \cdots \times (2n-2) \times (2n)}{2^{n}} =$ .....

(a) $\lfloor 2n$ (b) $\lfloor n$ (c) $n \lfloor n$ (d) $2 \lfloor n$

(45) If $X = \{x : x \in \mathbb{N}, 1 \le x \le 5\}$, $Y = \{(a, b) : a, b \in X, a \neq b\}$, then the number of elements in Y = ...

(a) 7 (b) 10 (c) 20 (d) 25

(46) $(n^2 + 3n + 2) \lfloor n = \lfloor \cdots$

(a) n^2 (b) $n+1$ (c) $n+2$ (d) $n(n+2)$

(47) If $n^2 + 9n + 20 = \frac{\lfloor n+5}{x}$, then x = ...

(a) $n + 3$ (b) $\lfloor n+3$ (c) $\lfloor n+4$ (d) ${}^{n+5}P_2$

(48) The solution set of the equation $\frac{\lfloor x}{10} = {}^{x-1}P_{x-3}$ is .....

(a) {5} (b) {6} (c) {7} (d) {8}

(49) $(n + 1)(n + 2)(n + 3) \times \cdots \times (2n)$ = ...

(a) $\lfloor 2n-1$ (b) $\frac{\lfloor 2n}{\lfloor n}$ (c) $\lfloor 3n+1$ (d) $\frac{\lfloor 2n}{\lfloor n+1}$

(50) If $\lfloor n < 100$, $\lfloor n+1 > 100$, then n = ...

(a) 4 (b) 5 (c) 6 (d) 7

(51) If $\lfloor n$ is a prime number , then n = ...

(a) zero (b) 1 (c) 2 (d) 3

(52) $n(n^2 - 1)(n^2 - 4)$ = ...

(a) ${}^{n-4}P_3$ (b) ${}^{n+2}P_3$ (c) ${}^{n+2}P_5$ (d) ${}^{n-2}P_5$

(53) $(\lfloor n)(\lfloor n)$ ..... $\lfloor n^2$

(a) > (b) ≤ (c) ≥ (d) =

(54) If ${}^nP_5 = {}^nP_4$, then n = ...

(a) 20 (b) 9 (c) 4 (d) 5

(55) If $2\,{}^nP_3 = {}^{n+1}P_3$, then n = ...

(a) 4 (b) 5 (c) 6 (d) 7

(56) If $\lfloor 7 = {}^7P_x$, then x = ...

(a) 6 or 7 (b) 7 (c) 1 or zero (d) 50 or 40

(57) If $\lfloor 5 \lfloor 9 = 12 \lfloor n$, then n = ...

(a) 10 (b) 11 (c) 12 (d) 13

(58) The highest common factor of the numbers $\lfloor n$, $\lfloor n+1$, $\lfloor n+2$ is .....

(a) $\lfloor n$ (b) $\lfloor n+2$ (c) n (d) $n + 2$

(59) The lowest common multiple of the numbers $\lfloor n$, $\lfloor n+1$, $\lfloor n+2$ is .....

(a) $\lfloor n$ (b) $\lfloor n+2$ (c) n (d) $n + 2$

(60) If a and b are two consecutive numbers where $a > b$, then a b = ...

(a) ${}^aP_2$ (b) ${}^bP_2$ (c) ${}^2P_a$ (d) ${}^2P_b$

(61) If $\lfloor 13 = 2^a \times b$ where b doesn't divisible by 2 , the a = ...

(a) 8 (b) 9 (c) 10 (d) 11

(62) If $0° \leq \theta < 360°$, then the solution set for the equation : $\lfloor \cos\theta = 1$

(a) $\{0° , 180°\}$ (b) $\{90° , 270°\}$

(c) $\{0° , 90° , 270°\}$ (d) $\{90° , 180°\}$

Second Essay questions :

1 Find the value of n which satisfies each of the following :

(1) $\lfloor n = 720$

(2) $^{15}P_n = 2730$

(3) $^nP_3 = 504$

(4) $^{(n-1)}P_4 = 120$

(5) $^nP_0 + {}^nP_1 + {}^nP_2 = 50$

(6) $\lfloor n - 4 = 120$

(7) $n \lfloor 2n - 1 = 12$

(8) $\frac{\lfloor n+1}{\lfloor n} = 5$

2 Find the solution set of each of the following equations :

(1) $\lfloor n - 5 = 1$ « $\{5 , 6\}$ »

(2) $\frac{\lfloor n+1}{\lfloor n-1} = 42$ « $\{6\}$ »

(3) $12 \times \lfloor n = \lfloor n + 2$ « $\{2\}$ »

(4) $\lfloor n = 30 \lfloor n - 2$ « $\{6\}$ »

3 Answer the following :

(1) If $^nP_4 = 14 \times {}^{n-2}P_3$ Find the value of : n « 7 or 8 »

(2) If $^5P_r = 2 \times {}^6P_{r-1}$ Find the value of : r « 3 »

(3) If $^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$ Find the value of : $\lfloor n - 3$ « 120 »

(4) If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$ Find the value of : n « 4 »

(5) If $^nP_r = 604800$, $\lfloor r = 5040$ Find the value of : n , r then find : $^{n+r}P_{n-r}$ « 10 , 7 , 4080 »

(6) If the middle factor in the expansion of $^nP_{11}$ equals 15 Find the value of : n « 20 »

4 Prove that :

(1) $\frac{\lfloor n+1}{\lfloor n-1} - \frac{\lfloor n}{\lfloor n-2} = 2n$

(2) $\frac{\lfloor n+3}{\lfloor n} = (n+3) \times {}^{n+2}P_2$

(3) $\frac{1}{\lfloor n} - \frac{1}{\lfloor n+1} + \frac{1}{\lfloor n+2} = \frac{n+1}{(n+2)\lfloor n}$

(4) $\frac{\lfloor 2}{\lfloor 6} + \frac{\lfloor 3}{\lfloor 7} + \frac{\lfloor 4}{\lfloor 8} = \frac{23}{\lfloor 7}$

(5) $^{n+1}P_{r+1} = (n+1)\,{}^nP_r$

(6) $\frac{^nP_r}{\lfloor r} = \frac{n}{r} \times \frac{^{n-1}P_{r-1}}{\lfloor r-1}$

(7) $^nP_r : {}^{n-1}P_{r-1} = n$, then deduce the value of : $^{97}P_{40} : {}^{96}P_{39}$ « 97 »

5 **Find the value of n if :**

(1) $\frac{1}{\lfloor n} + \frac{2}{\lfloor n+1} = \frac{56}{\lfloor n+2}$ « 5 »

(2) $\frac{\lfloor n+1}{\lfloor n+2} + \frac{\lfloor n}{\lfloor n+1} = \frac{13}{42}$ « 5 »

---

6 **Find :**

(1) The number of the different ways , for 5 students to sit on 7 seats in one row. « 2520 »

(2) How many ways can 9 persons be arranged in a circle ? « 40320 »

(3) The number of ways to select a president , vice president and a secretary out of ten person committee. « 720 »

(4) How many ways can Hossam have a meal out of three meals (kofta - chicken - fish) and a drink out of two drinks (juice - soft drink) ? Represent it using the tree diagram. « 6 »

(5) How many permutations can be formed from five different letters from the alphabetic ? « 7893600 »

(6) How many different three digit numbers can be taken so that the unit digit from $\{3 , 7\}$, tens digit from $\{2 , 4 , 9\}$, hundreds digit from $\{1 , 5\}$ « 12 »

(7) How many ways can a different two-digit number be formed from the numbers $1 , 2 , 3 , 4$ « 12 »

(8) How many ways can a three-digit number be formed from the numbers $\{2 , 3 , 5\}$ « 27 »

(9) How many ways can a two-digit number be formed from the numbers $1 , 2 , 3 , 4$? « 16 »

(10) How many ways can a different three-digit number be formed from the numbers $\{0 , 1 , 2 , 3 , 4\}$ « 48 »

(11) How many different three-digit even numbers can be formed from the set of the numbers $\{2 , 3 , 4 , 5 , 7\}$ « 24 »

(12) How many different four-digit numbers can be formed from the elements $\{2 , 3 , 4 , 7\}$ so that the tens digit is even. « 12 »

---

7 **By how many ways a number can be formed from the digits $3 , 4 , 5 , 6 , 7 , 8 , 9$:**

(1) If the number formed from 3 different digits. « 210 »

(2) If the number formed from all the digits without repeating any digit. « 5040 »

(3) If the number formed from 5 different digits and divisible by 2 « 1080 »

(4) If the number formed from 4 different digits and its unit digit is 7 « 120 »

(5) If the number formed from 4 different digits and less than 6000 « 360 »

8 If $X = \{2, 3, 5, 7, 9\}$:

(1) How many ways can a different two-digit number be formed from these numbers ? « 20 »

(2) How many ways can a two-digit number be formed from these numbers ? « 25 »

(3) How many ways can a different three-digit number be formed from these numbers ? « 60 »

(4) How many ways can a different three-digit number less than 500 be formed from these numbers ? « 24 »

(5) How many ways can a different four-digit number whose unit digit is 2 be formed from these numbers ? « 24 »

9 From the set of the letters $\{a, b, c, d, e, f\}$, find :

(1) Number of ways to select a letter. « 6 »

(2) Number of ways to select two letters with considering the order. « 30 »

10 If $X = \{x : x \text{ is a natural number }, 1 \leq x \leq 7\}$, $Y = \{(a, b) : a, b \in X, a \neq b\}$:
Find the number of elements of each of : X , Y « 7 , 42 »

11 If ${}^{x+y}P_3 = 504$, ${}^{x-y}P_4 = 120$ Find the value of : ${}^{x}P_y$ « 42 »

12 Solve each of the following equations :

(1) ${}^{x+2}P_3 = 42\,x$ « 5 »

(2) ${}^{7}P_x - x \times {}^{7}P_{x-1} = 0$ « 4 »

(3) $\lfloor x - 3 \times {}^{x}P_3 = 20 \lfloor x - 2$ « 5 »

(4) $\lfloor x + 2 = 182 \times {}^{x}P_{11} \lfloor x - 11$ « 12 »

13 Find the solution set of the equation : $30 \times {}^{x+3}P_3 = {}^{x+5}P_5$ if $x \in \mathbb{N}$ « {1} »

14 If ${}^{n}P_{15} > {}^{n}P_{14}$ Find the smallest value of the number : n satisfying the inequality « 16 »

15 If ${}^{a+b}P_4 = 840$, $\lfloor a - b = 6$ Find the value of each of : a , b , $\lfloor 2a - 5b$ « 5 , 2 , 1 »

16 If ${}^{n}P_r = 2520$, $\lfloor r = 120$ Find : $\lfloor 3r - 2n$ « 1 »

17 Prove that :

(1) $\lfloor 80 = 2^{40} \lfloor 40\,(1 \times 3 \times 5 \times \ldots \times 79)$

(2) $\dfrac{\lfloor 100}{\lfloor 50} = 2^{50}\,(1 \times 3 \times 5 \times \ldots \times 99)$

(3) $\lfloor 2n = 2^n \lfloor n \left(1 \times 3 \times 5 \times \ldots (2n-1)\right)$

18 Prove that :

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} + \frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1} = \frac{\lfloor n+1}{\lfloor r \lfloor n-r+1}$$

Third Higher skills

Choose the correct answer from those given :

(1) If ${}^{n}P_{2}$, ${}^{n}P_{3}$, ${}^{n+1}P_{3}$ form an arithmetic sequence , then $n + \lfloor n - 3 = $ .....

(a) 4 (b) 5 (c) 12 (d) 31

(2) The number of solutions for the equation : $x = \lfloor x$ in $\mathbb{Z}$ equals .....

(a) zero (b) 1 (c) 2 (d) infinite solutions.

(3) If $\lfloor n$ is divisible by both 7 and 13 , then .....

(a) $n \leq 7$ (b) $n = 10$ (c) $7 \leq n \leq 13$ (d) $n \geq 13$

(4) If the unit digit in the number $\lfloor n$ is not equal to zero , then .....

(a) $n > 4$ (b) $n < 5$

(c) $n > 9$ (d) n is an odd number.

(5) If $\lfloor r = n$, $\lfloor n = r$, then $r + n =$ .....

(a) 2 (b) zero or 1 (c) 2 or 3 (d) 2 or 4

(6) If the side lengths of a triangle are $\frac{1}{2}\lfloor n$, $\lfloor n - 2$, $\lfloor 2 - n$ cm. , then the numerical value of the area of the triangle = ... cm^2

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{8}$ (d) $\frac{\sqrt{3}}{16}$

(7) The remainder when you divide $\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + ... + \lfloor 100$ by 7 is .....

(a) 0 (b) 3 (c) 5 (d) 6

(8) If $\lfloor n$, $3\lfloor n$, $\lfloor n + 1$ form a geometric sequence , then .....

(a) $\lfloor n$, $4\lfloor n$, $\lfloor n + 1$ form a geometric sequence.

(b) $\lfloor n$, $4\lfloor n$, $\lfloor n + 1$ form an arithmetic sequence.

(c) $\lfloor n$, $5\lfloor n$, $\lfloor n + 1$ form a geometric sequence.

(d) $\lfloor n$, $5\lfloor n$, $\lfloor n + 1$ form an arithmetic sequence.

(9) The unit digit in the number $\lfloor 0 + \lfloor 1 + \lfloor 2 + \lfloor 3 + ... + \lfloor 2022$ is .....

(a) zero (b) 4 (c) 6 (d) 9

(10) If a is a natural number such that $x = \lfloor a + \lfloor a + 1 + \lfloor a + 2$, then the integer b satisfies $\frac{x}{b} = \lfloor a$ is .....

(a) always even. (b) always odd. (c) prime number. (d) a perfect square.

Life applications

1 An Ice-cream shop offers three size (small - medium - large) and five flavors (strawberry , mango , lemon , milk , chocolate) How many ways are available to buy one of these sizes by one of these flavors ?

« 15 »

2 To make a secret number of a money box formed from 4 numbers except zero , How many ways to form this number ? « 6561 »

3 A phone number is made up of 8 places

9	c	....	....	....	....	....	....

c should be a number of 3 , 4 , 5 , 8 while the rest of the digits made up of any unconditional digit. How many different telephone numbers are available ? « 4000000 »

4 If you know that the set of the numbers of mobile networks in a country is made up of an eleven-digit number. If the number (025) is constant on left side , find the greatest number of phone lines which the mobile network can stand. « 100000000 »

5 The licence plates of cars in a governorate start with three letters followed by three digits except zero. How many plates can be got assuming that there is no repetition for any letter or digit in the licence plates ? « 7862400 »

Exercise

8

Combinations

From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) ${}^{9}C_{1} + {}^{100}C_{0} + {}^{5}C_{5} =$ .....

(a) 1 (b) 9 (c) 11 (d) 114

(2) If $|3n - 7| = 120$, then ${}^{n}C_{n-1} =$ .....

(a) 3 (b) 4 (c) 5 (d) 6

(3) If $|x - 3| = 1$, then ${}^{x+2}C_{x-1} =$ .....

(a) 3 or 4 (b) 9 or – 2 (c) 10 or 20 (d) 9 or 10

(4) If ${}^{n}C_{n-2} = 36$, then n = ...

(a) 6 (b) 9 (c) 12 (d) 24

(5) If ${}^{n}C_{3} = 120$, then ${}^{n}C_{n-8} =$ .....

(a) 90 (b) 60 (c) 45 (d) 30

(6) If ${}^{2n-1}C_{3} = 84$, then $|n - 5| =$ .....

(a) 1 (b) 2 (c) 6 (d) 24

(7) If ${}^{8}P_{r+1} = 336$, then ${}^{2r}C_{4} =$ .....

(a) 16 (b) 4 (c) 2 (d) 1

(8) If ${}^{n+1}P_{3} = 24 \times {}^{n-1}C_{2}$, then n = ...

(a) 3 only. (b) 5 only. (c) 8 only. (d) 3 or 8

(9) If ${}^{m+n}P_{2} = 380$, ${}^{n-1}C_{n-2} = 4$, then $\frac{m}{n} =$ .....

(a) 3 (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{4}$

(10) If ${}^{n}P_{n-3} = 20$, ${}^{m}C_{n} = {}^{m}C_{2n+1}$, then $m \times n =$ .....

(a) 20 (b) 40 (c) 60 (d) 80

(11) If ${}^{6}C_{x} = {}^{4}C_{x}$, then $x =$ .....

(a) zero (b) 1 (c) 2 (d) 3

(12) If ${}^{40}C_{4} = \frac{\lfloor 40}{\lfloor 4 \lfloor x}$, then $x =$ .....

(a) 4 (b) 44 (c) 36 (d) 40

(13) If ${}^{n}P_{r} = \lfloor r \times {}^{n}C_{x}$, then x can be equals .....

(a) n (b) n – r (c) n + r (d) r – n

(14) If ${}^{m}P_{n} = 24\ {}^{m}C_{n}$, then n = ...

(a) 1 (b) 3 (c) 4 (d) 6

(15) If ${}^{15}C_{r} = {}^{15}C_{6}$, then r = ...

(a) 6 (b) 9 (c) 8 (d) 6 or 9

(16) If $\lfloor x = {}^{5}C_{2} \times {}^{9}P_{2}$, then $x =$ .....

(a) 6 (b) 9 (c) 10 (d) 120

(17) If ${}^{6}C_{r} > 1$, ${}^{r}C_{4} > 1$, then $\lfloor r =$ .....

(a) 6 (b) 24 (c) 120 (d) 720

(18) If ${}^{n}P_{r} = {}^{n}C_{r}$, then r = ...

(a) zero (b) 1 (c) zero or 1 (d) 1 or 2

(19) If ${}^{n}P_{1} + {}^{n}C_{2} = 36$, then n = ...

(a) 9 (b) – 9 or 8 (c) 8 (d) 9 or – 8

(20) If ${}^{12}C_{r^2} = {}^{12}C_{2r^2}$, then r ∈ .....

(a) $\{0, 2\}$ (b) $\{0, -2\}$ (c) $\{2, -2\}$ (d) $\{0, 2, -2\}$

(21) If ${}^{n}P_{2} = 42$, $\lfloor n - r = 120$, then ${}^{n}C_{r+3} =$ .....

(a) 2 (b) 7 (c) 21 (d) 42

(22) If $\frac{{}^{n}P_{3}}{{}^{n-1}P_{2}} = {}^{n}C_{2}$, then n = ...

(a) 3 (b) 4 (c) 5 (d) 4 or 3

(23) If ${}^{n+1}C_{n-2} = 56$, then n = ...

(a) 5 (b) 6 (c) 7 (d) 8

(24) If ${}^{n}C_{3} + {}^{n}C_{n-3} = 2\,{}^{n}C_{4}$, then n = ...

(a) 8 (b) 7 (c) 2 (d) 11

(25) ${}^{n}P_{r} : {}^{n}C_{r}$ = ...

(a) $\lfloor n-1$ (b) $\lfloor r$ (c) $\lfloor n$ (d) 1

(26) If ${}^{13}C_{r+2} : {}^{13}C_{r+1} = 9 : 5$, then r = ...

(a) 2 (b) 3 (c) 4 (d) 5

(27) If ${}^{n-m}P_{3} = 210$, ${}^{n+m}C_{4} = 715$, then m × n = ...

(a) 15 (b) 30 (c) 35 (d) 50

(28) ${}^{n}P_{r} = 120 \times {}^{n}C_{n-r}$, then r = ...

(a) 2 (b) 3 (c) 4 (d) 5

(29) If ${}^{n}C_{10} = {}^{n}C_{14}$, then ${}^{25}C_{n}$ = ...

(a) 24 (b) 25 (c) 1 (d) 49

(30) If ${}^{27}C_{2r+1} = {}^{27}C_{5r-8}$, then ${}^{5}C_{r}$ = ...

(a) 10 (b) 12 (c) 13 (d) 15

(31) If ${}^{n}C_{10} > {}^{n}C_{9}$, then n .....

(a) = 19 (b) > 19 (c) < 19 (d) ≤ 19

(32) If ${}^{2n+1}C_{2n-1} - 2 \times {}^{n+2}C_{n} = 46$, then n = ...

(a) 5 (b) 6 (c) 7 (d) 8

(33) The smallest value of the number (n) makes ${}^{n}P_{r-2} = 24\,{}^{n}C_{r-2}$ is .....

(a) 4 (b) 5 (c) 6 (d) 8

(34) If ${}^{4}C_{0} + {}^{4}C_{1} + {}^{4}C_{2} + {}^{4}C_{3} + {}^{4}C_{4} = 2^{n}$, then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

(35) ${}^{1}C_{0} + {}^{2}C_{1} + {}^{3}C_{2} + \ldots + {}^{n}C_{n-1}$ = ...

(a) $\frac{n^2}{2}$ (b) $\frac{n^2+n}{2}$ (c) $\frac{n^2-n}{3}$ (d) $\frac{n^2+2n}{2}$

(36) If ${}^{n}C_{5} \leq {}^{n}C_{4}$, then n ∈ .....

(a) $\{5, 4\}$ (b) $\{4, 9\}$

(c) $\{5, 6, 7, 8, 9\}$ (d) $\{4, 5, 6, 7, 8, 9\}$

(37) If ${}^{n}C_{r} = {}^{n}C_{2r}$ where : $n , r \in \mathbb{Z}^{+}$, then n multiple of .....

(a) 2 (b) 3 (c) 4 (d) 5

(38) If ${}^{x+y}P_{4} = 360$, $\lfloor 2x+y = 5040$, then ${}^{y}C_{2x} =$ .....

(a) 6 (b) 8 (c) 10 (d) 12

(39) If ${}^{n}C_{r} : {}^{n}C_{r-1} = 7 : 4$, ${}^{n}P_{r} : {}^{n-1}P_{r} = 5 : 3$, then n + r = ...

(a) 7 (b) 8 (c) 11 (d) 14

(40) If ${}^{n^2+n}C_{3} = {}^{n^2+n}C_{9}$, then n may be equals .....

(a) 12 (b) 6 (c) 3 (d) 2

(41) If ${}^{n}C_{3}$, ${}^{n}P_{3}$, $\lfloor n$ form a geometric sequence , then n = ...

(a) 5 (b) 6 (c) 8 (d) 12

(42) ${}^{7}C_{0} + {}^{7}C_{2} + {}^{7}C_{3} + {}^{7}C_{4} + {}^{7}C_{5} + {}^{7}C_{6} + {}^{7}C_{7} = 2^{7} +$ .....

(a) 1 (b) – 1 (c) – 7 (d) 7

(43) The number of permutations could be formed together from (n) elements equals .....

(a) $n \lfloor n$ (b) ${}^{n+1}P_{n}$ (c) ${}^{n}C_{n}$ (d) ${}^{n}C_{n} \lfloor n$

(44) If the number of ways of selecting 3 elements together from n elements equals 10 , then n = ...

(a) 30 (b) 10 (c) 6 (d) 5

(45) If 4 friends meet each other and each shakes hands only once with each of the others , how many hands shakes will there be ?

(a) 16 (b) 8 (c) 6 (d) 4

(46) 7 people have participated in a chess game so that a game is held between each two players , then the number of matches is .....

(a) 42 (b) 28 (c) 21 (d) 18

(47) The number of diagonals in the octagon is .....

(a) 8 (b) 20 (c) 32 (d) 18

(48) The number of sides of a polygon which has 44 diagonals is .....

(a) 7 (b) 8 (c) 11 (d) 12

(49) The number of ways can 4 elements be chosen together from 10 elements regardless the order is .....

(a) $(10)^{4}$ (b) ${}^{10}P_{4}$ (c) ${}^{10}C_{4}$ (d) $\lfloor 4$

(50) The number of ways can 7 students be selected from 10 students to take a historical trip is .....

(a) 80 (b) 120 (c) 144 (d) 70

(51) The number of ways to select a red ball and a white ball from 6 red balls numbered from 1 to 6 and 8 white balls numbered from 1 to 8 is .....

(a) 2 (b) 14 (c) 24 (d) 48

(52) If the number of ways to select 3 elements together from a set equals the number of ways to select 5 elements together from the same set , then the number of elements of this set equals .....

(a) ${}^5C_3$ (b) ${}^5P_3$ (c) 8 (d) 15

(53) The number of boys in a class is twice the number of girls , if the number of ways to select a boy and a girl is 72 , then the number of boys equals .....

(a) 4 (b) 6 (c) 12 (d) 18

(54) How many ways can a committee of two men and a woman be selected from 7 men and 5 women ?

(a) 210 (b) 105 (c) 26 (d) 75

(55) It is wanted to choose a teacher out of 4 teachers to train the olympics students in mathematics and another teacher to prepare the test , the number of ways of choosing = ... ways.

(a) 16 (b) 8 (c) 12 (d) 15

(56) In a football league , each two teams play together once , if the number of matches in this league is 153 mathces , then the number of the competing teams equals .....

(a) 9 (b) 13 (c) 18 (d) 19

(57) The number of ways can a person choose one kind or more of fruit from 5 kinds of fruits is .....

(a) 32 (b) 31 (c) 325 (d) 63

(58) Five coplanar points , if there is no 3 points of these points lie on the same line , then the number of triangles can be formed from these points is .....

(a) 5 + 3 (b) 5 × 3 (c) ${}^5P_3$ (d) ${}^5C_3$

(59) If the points A , B , C , D , E and F lie on a circle , then the number of line segments can be drawn to join these points is .....

(a) ${}^6P_2$ (b) ${}^5P_2$ (c) ${}^6C_2$ (d) ${}^5C_2$

(60) The number of coplanar quadrilaterals can be formed by joining some vertices of a hexagon = ...

(a) 15 (b) 20 (c) 24 (d) 30

(61) By how many ways 8 toys can be distributed among 3 children such that the first child takes 3 toys and the second takes two toys and the third takes the remainder ?

(a) ${}^8C_3 + {}^8C_2 + {}^8C_8$ (b) ${}^8C_3 \times {}^8C_2$ (c) ${}^8C_3 \times {}^5C_2 \times {}^5C_3$ (d) ${}^8C_3 \times {}^5C_2 \times {}^3C_3$

(62) An exam consists of 6 questions , a student needs to answer correctly at least 3 questions to pass the exam. The number of ways can a student pass the exam is .....

(a) 20 (b) 1800 (c) 15 (d) 42

(63) An exam consists of 6 questions , 2 are compulsory and 4 are optional. A student needs to answer 3 or more questions to pass the exam , then the number of ways can a student pass the exam is .....

(a) 42 (b) 96 (c) 15 (d) 48

Second Essay questions

1 **Write in terms of permutations each of :**

(1) ${}^8C_3$ (2) ${}^{19}C_2$ (3) ${}^5C_0$ (4) ${}^xC_{x-y}$

2 **Use the form ${}^nC_r$ to write each of :**

(1) $\frac{{}^8P_2}{\lfloor 2}$ (2) $\frac{{}^9P_3}{\lfloor 3}$ (3) $\frac{{}^{10}P_4}{\lfloor 4}$ (4) $\frac{{}^8P_0}{\lfloor 0}$

3 If ${}^mC_{m-4} = 35$, find the value of : m and if ${}^nC_m = {}^nC_{m+1}$ find the value of : n « 7 , 15 »

4 If ${}^nC_3 = 35$, ${}^xC_n = {}^xC_{n+3}$, what is the value of : x « 17 »

5 If ${}^{28}C_r = {}^{28}C_{2r-47}$, find the value of : r « 25 »

6 If ${}^{25}C_{2r+1} = {}^{25}C_{3r-1}$, what is the value of : r « 2 or 5 »

7 If ${}^{36}C_{3r-5} = {}^{36}C_{r-3}$, find the value of : ${}^{11}C_r$ « 1 »

8 If ${}^{n+1}C_4 = \frac{5}{2}\,{}^nC_3$, find the value of : n « 9 »

9 If ${}^nC_3 = 30\frac{1}{3}\,n$, find the value of : n « 15 »

10 If ${}^{n}C_{4} : {}^{n-2}C_{3} = 7 : 2$, what is the value of : n « 7 or 8 »

11 If ${}^{n}C_{5} : {}^{n}C_{4} = 8 : 5$, what is the value of : n « 12 »

12 **Find the solution set in each of the following :**

(1) ${}^{n}C_{3} = 84$ «{9}»

(2) ${}^{12}C_{r} = {}^{12}C_{2r+3}$ «{3}»

(3) ${}^{n}C_{2n-21} = {}^{n}C_{3}$ «{12 , 18}»

(4) ${}^{n+1}C_{3} = 4\,{}^{n-1}C_{2}$ «{8 , 3}»

13 **Find the value of each of the following :**

(1) ${}^{5}C_{0} + {}^{5}C_{1} + {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$ «32»

(2) ${}^{5}C_{0} - {}^{5}C_{1} + {}^{5}C_{2} - {}^{5}C_{3} + {}^{5}C_{4} - {}^{5}C_{5}$ «0»

14 If ${}^{2n+m}C_{2} = 190$, ${}^{n-2m}P_{3} = 60$, find the value of each of : n , m « 9 , 2 »

15 If ${}^{n}P_{r-1} = 6720$, ${}^{n}C_{r-1} = 56$, what is the value of : n , r « 8 , 6 »

16 **Prove that :**

${}^{n}C_{r} \div {}^{n-1}C_{r-1} = \frac{n}{r}$, then deduce the value of : ${}^{100}C_{50} \div {}^{99}C_{49}$ « 2 »

17 **Prove that :**

${}^{n}C_{r} \div {}^{n}C_{r-1} = \frac{n-r+1}{r}$, then deduce the value of : ${}^{10}C_{6} \div {}^{10}C_{5}$ « $\frac{5}{6}$ »

18 **Prove that :**

${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$, then deduce the value of : ${}^{10}C_{3} + {}^{10}C_{4}$ « 330 »

19 **Prove that :** ${}^{2n}C_{n} = 2^{n} \times \frac{1 \times 3 \times 5 \times \ldots\ldots \times (2n-1)}{\lfloor n}$

20 If ${}^{2n+1}P_{n-1} : {}^{2n}C_{n+1} = \frac{3}{2} \lfloor n-1$, what is the value of : n « 4 »

21 Find the value of x , y in $\mathbb{Z}^{+}$ from the two equations :

${}^{x}C_{y} = 2 \times {}^{x}C_{y+1}$, ${}^{x}C_{y} = {}^{x}C_{y-1}$ « $x = 5$, $y = 3$ »

22 **Prove that :** $\frac{{}^{n}C_{1} \times {}^{2n}C_{2}}{3\,{}^{n}C_{3}} = \frac{{}^{2n}P_{2}}{3\,{}^{n-1}P_{2}}$

23 **Prove that :** $\lfloor 25$ is divisible by $\lfloor 13 \lfloor 12$

24 **Answer the following :**

(1) If 3 students are selected from (n) number of students to attend a seminar so that the number of ways of choosing is 10 ways , find the number of students. « 5 »

(2) How many ways can a five-student activity committee formed from three male students and two female students be selected out of a class contains 10 male students and 8 female students ? « 3360 »

(3) How many ways can a committee of 4 men or 3 women be selected out of 6 men and 5 women ? « 25 »

(4) How many ways can a five member team and a captain be selected out of 10 students practicing basketball. « 1260 »

25 How many ways can 2 committees each one formed from 3 persons be selected from 12 persons such that no one is chosen in the 2 committees. « 18480 »

26 If $X = \{x : x \in \mathbb{N} , 5 \leq x \leq 9\}$, $Y = \{(a , b) : a , b \in X , a \neq b\}$, $Z = \{\{a , b , c\} : a , b , c \in X\}$, find the number of elements of Y , Z « 20 , 10 »

27 How many ways can a teacher choose a student or more from 6 students? « 63 »

28 How many ways can a five people committee take the majority of a decision ? « 16 »

29 From a class of 7 boys and 6 girls it is required to select a team containing 5 persons , find the number of ways in which the team is selected if the members of the team :

(1) Are of any sex.

(2) Are only boys.

(3) Are only girls.

(4) Are of the same sex.

(5) Has 3 boys and 2 girls. « 1287 , 21 , 6 , 27 , 525 »

30 A university student learns different eight subjects and he cannot join the next grade till he succeeds in six subjects at least. How many ways can the student join the next grade ? « 37 »

31 A box contains 6 white balls and 4 red balls , find the number of ways in which can select 3 balls at a time if :

(1) The selected balls from any colour.

(2) The selected balls contains 2 white balls exactly.

(3) The selected balls contains 2 white balls at least.

(4) The selected balls contains 2 white balls at most. « 120 , 60 , 80 , 100 »

32 9 persons are nominated to choose 3 of them as ambassadors for an arabian country. How many ways of choosing ? and if a certain person must be selected in any choice , find in how many ways they are selected. If a certain person must be set aside, find in how many ways they are selected ?

« 84 , 28 , 56 »

Third Higher skills

Choose the correct answer from those given :

(1) ${}^{6}C_{r}$ has maximum value , when r = ...

(a) 1 (b) 2 (c) 3 (d) 6

(2) If ${}^{n}P_{x} + {}^{x}P_{n} = 1440$, then ${}^{n+4}C_{x-5}$ = ...

(a) 6 (b) 9 (c) 10 (d) 12

(3) If ${}^{4}C_{r+2} = {}^{4}C_{2-r}$, then r ∈ .....

(a) $\{0\}$ (b) $\{-1 , 4\}$ (c) $\{0 , 3\}$ (d) $\{-2 , -1 , 0 , 1 , 2\}$

(4) $\sum_{r=0}^{n} {}^{n}C_{r}$ = ...

(a) 2^{r} (b) n^{2} (c) 2^{n} (d) $1 - r^{2}$

(5) If the points A , B , C , D , E , F lie on a circle , then the number of polygons can be drawn joining these points = ...

(a) 20 (b) 15 (c) 30 (d) 42

(6) If $m = {}^{n}C_{2}$, then ${}^{m}C_{2}$ = ...

(a) $3 \times {}^{n}C_{3}$ (b) $4 \times {}^{n+1}C_{3}$ (c) $4 \times {}^{n}C_{4}$ (d) $3 \times {}^{n+1}C_{4}$

(7) The number of parallelograms can be formed from (m) parallel straight lines intersect (n) parallel straight lines equals .....

(a) ${}^{m-1}C_{2} \times {}^{n-1}C_{2}$ (b) ${}^{m-1}C_{2} \times {}^{n}C_{2}$

(c) ${}^{m}C_{2} \times {}^{n-1}C_{2}$ (d) ${}^{m}C_{2} \times {}^{n}C_{2}$

Second

Calculus and Trigonometry

UNIT 3

Calculus.

UNIT 4

Trigonometry.

Unit Three

Calculus

Exercise 9 Rate of change.

Exercise 10 Differentiation.

Exercise 11 Rules of differentiation.

Exercise 12 Derivative of composite function (Chain rule).

Exercise 13 Derivatives of trigonometric functions.

Exercise 14 Applications on derivatives.

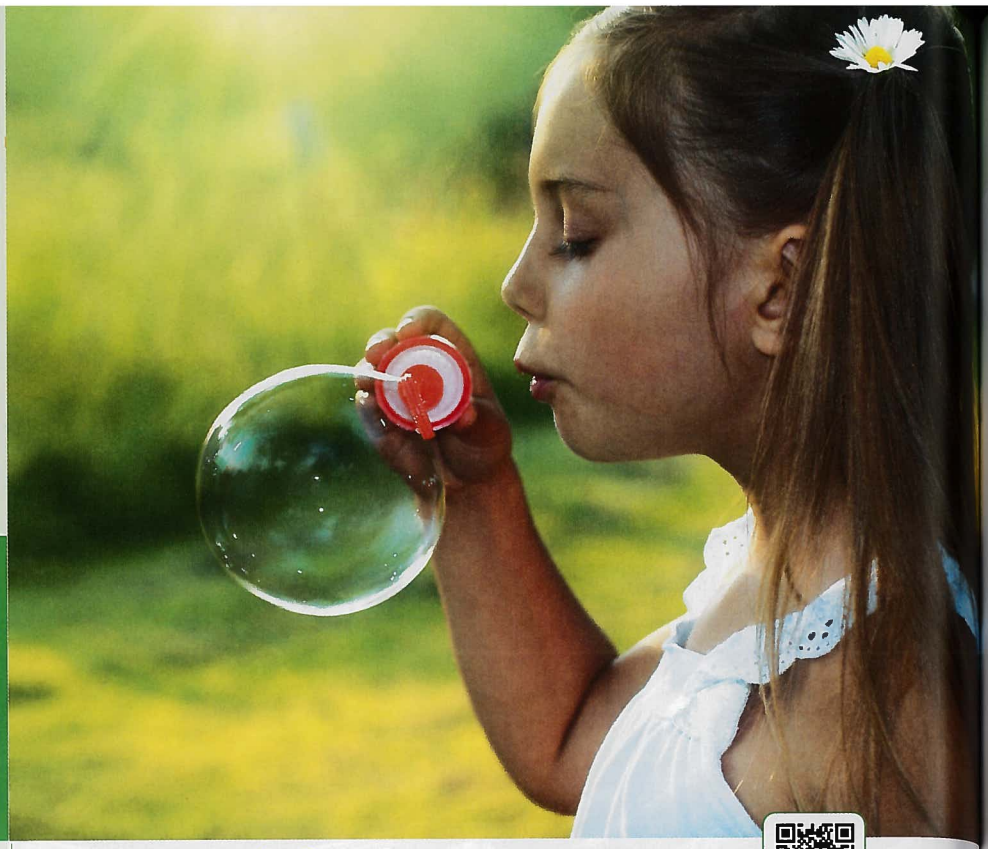
Exercise 15 Integration.

Scan the QR code to solve an interactive test on each lesson

Exercise

9

Rate of change



From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If the function $f : f(x) = 3x - 2$, then the variation function v (h) = ... at $x = 1$

(a) 3 (b) h (c) 3 h (d) 6 h

(2) If $f(x) = 4x + 1$, then the variation in f when x changes from 2 to 2.1 equals .....

(a) 0.1 (b) 0.4 (c) 4 (d) 4.1

(3) If $f(x) = x^2 + 2x + 3$, then v (h) = ... at $x = 2$

(a) $h^2 + 2h + 3$ (b) $h^2 + 6h + 6$ (c) $2h - 1$ (d) $h^2 + 6h$

(4) If $f(x) = x^2 - x + 1$, then :

First : The variation function v when $x = 3$ is .....

(a) v (3) (b) v $(x + 3)$ (c) $2h - 1$ (d) $h^2 + 5h$

Second : v (0.2) = ...

(a) 1.04 (b) 0.84 (c) 1.08 (d) 1.82

Third : v (– 0.3) = ...

(a) 1.41 (b) 1.31 (c) 1.04 (d) – 1.41

(5) The average rate of change of the function f where $f(x) = x^2$ when x varies from 3 to 3.1 equals .....

(a) 0.61 (b) 6.1 (c) 9 (d) 9.61

(6) The function $f : f(x) = x^2 - x$, so the average rate of change when x increases by 0.3 is .....

(a) $2x - 1$ (b) $2x - 0.7$ (c) -0.21 (d) -0.1

(7) The average rate of change of the function $f : f(x) = \frac{1}{x}$ when x changes from (x_1) to $(x_1 + h)$ equals .....

(a) $\frac{-1}{x_1}$ (b) $\frac{-h}{x_1(x_1+h)}$ (c) $\frac{-1}{x_1(x_1+h)}$ (d) $\frac{h}{x_1(x_1+h)}$

(8) If the average rate of change in f equals 2.4 when x varies from 3 to 3.2 , then the change in f equals .....

(a) 0.32 (b) 0.48 (c) 3.6 (d) 7.2

(9) If the average rate of change in f equals 5 when x varies from 2 to 4 and $f(2) = 6$, then $f(4) =$ .....

(a) -4 (b) 7 (c) 8 (d) 16

(10) If f is a function and the variation in f equals 14 when x changes from 2 to 4 , then the average rate of change in f equals .....

(a) 14 (b) 7 (c) $\frac{7}{2}$ (d) -7

(11) If the curve of function f passes through the two points $(1 , 2) , (2 , 3)$, then the average rate of change of the function f where x changes from 1 to 2 is .....

(a) 1 (b) 2 (c) zero (d) -1

(12) If $f(2) = 5$, $f(2.3) = 7$, then the variation of f when x changes from 2 to 2.3 equals .....

(a) 0.3 (b) 2 (c) 35 (d) 12

(13) If $f : f(x) = x^2 + ax - 3$ and the variation function v , $v\left(\frac{1}{2}\right) = \frac{19}{4}$ when $x = 2$, then $a =$ .....

(a) 3 (b) 4 (c) 5 (d) 6

(14) The radius length of a circle is r , then the average rate of change in the area of the circle when r changes from (r_1) to $(r_1 + h)$ is.....

(a) $2\pi r_1$ (b) $\pi(2r_1 + h)$ (c) πr_1^2 (d) $\pi(2r_1 + h^2)$

(15) The radius length of a circle is r , then the average rate of change in the circumference of the circle when r changes from r_1 to r_2 is .....

(a) $2\pi r_1$ (b) $2\pi(r_2 - r_1)$ (c) πr_1 (d) 2π

(16) The rate of change of the function $f : f(x) = x^2 + 5x - 2$ when $x = -1$ equals .....

(a) 1 (b) 2 (c) 3 (d) 4

(17) The rate of change of the function $f : f(x) = \frac{1}{x}$ at $x = \sqrt{5}$ equals .....

(a) 5 (b) $\frac{1}{5}$ (c) -5 (d) $-\frac{1}{5}$

(18) The rate of change of the function $f : f(x) = \sqrt{x}$ where $x \geq 0$ at $x = 16$ equals .....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

(19) If $f : f(x) = x^4$, then the rate of change at $x = 3$ equals .....

(a) 81 (b) 27 (c) 108 (d) 324

(20) If $f : f(x) = \cos x$, then the rate of change at $x = \frac{\pi}{2}$ is .....

(a) ± 1 (b) -1 (c) 1 (d) zero

(21) If $f : f(x) = \frac{3}{x-2}$, then :

First : The average rate of change of the function f = ...

(a) $\frac{-3h}{(x_1 - 2)(x_1 + h - 2)}$ (b) $\frac{-3}{(x_1 - 2)(x_1 + h - 2)}$

(c) $\frac{-3}{2(3+h)}$ (d) $\frac{3}{h-2}$

Second : The rate of change of f at $x = 5$ is .....

(a) -3 (b) $-\frac{1}{3}$ (c) 5 (d) $-\frac{1}{5}$

(22) If $f(x) = a$, where a is a constant , then the average rate of change of the function f is .....

(a) a (b) $-a$ (c) zero (d) 1

(23) If $f(x) = ax + b$, then the average rate of change of the function f when x changes from x_1 to x_2 is .....

(a) $-a$ (b) a (c) $a + b$ (d) 1

(24) If the average rate of change in the function f where $f(x) = 4x^2 + a$ equals -4 when x changes from b to 2 , then b = ...

(a) -2 (b) -3 (c) -4 (d) 20

(25) The average rate of change in the cube volume with respect to its edge length when its edge length varies from 5 cm. to 7 cm. equals .....

(a) 125 (b) 343 (c) 218 (d) 109

(26) Lamina in the shape of a square expanding regularly reserving its shape , then the rate of change of its area with respect to its side length when its side length 5 cm. equals .....

(a) 10 (b) 5 (c) 25 (d) 100

(27) A spherical ballon is expanding because of the pressure inside it such that it preserves its shape , then the average rate of change of its surface area with respect to its radius length when its radius length changes from 7 cm. to 9 cm. equals .....

(a) 16π (b) 32π (c) 64π (d) 128π

(28) The size of a bacterial culture at any time t (measured in minute) is given by the relation $f(t) = 2t^3 + 100$ mg. , then the rate of the instantaneous growth of the function f when $t = 5$ is .....

(a) 250 (b) 125 (c) 105 (d) 150

(29) A triangular lamina whose base length equals twice its corresponding height. It expands by heating maintaining its shape , then the average rate of change in its area with respect to its height if its height varies from 8 cm. to 8.4 cm. equals .....

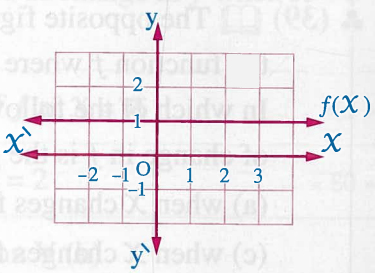
(a) 15.8 (b) 16.4 (c) 14.2 (d) 18.6

(30) The length of a rectangular lamina is 3 times its width. If expands and preserves its shape and the ratio between its dimensions , then the rate of change in its area with respect to length when its length = 6 cm. equals .....

(a) 4 (b) 6 (c) 8 (d) 12

(31) In the given figure :

The average rate of change of the function f when X changes from -1 to 2 equals .....



(a) zero (b) -1

(c) 2 (d) 3

(32) In which of the following function , the average rate of change of the function when X changes from (X_1) to $(X_1 + h)$ is constant ?

(a) $f(X) = 4X^2 + 10$ (b) $f(X) = 2X - 3$

(c) $f(X) = \frac{1}{2}X^3 + 7X$ (d) $f(X) = \frac{1}{X}$

(33) In which of the following functions , has variation of f when X changes from a to $a + h$ is equal to the variation of f when X changes from $a - h$ to a ?

(a) $f(X) = X^2$ (b) $f(X) = 3X + 2$ (c) $f(X) = X^3$ (d) $f(X) = \cos X$

(34) If $f(X) = \begin{cases} X^2 & , \quad X < 4 \\ 3 - X & , \quad X \geq 4 \end{cases}$, then the rate of change of f when $X = 7$ is .....

(a) 3 (b) -1 (c) -2 (d) $\frac{-1}{3}$

(35) If f is an even function , then the average rate of change of the function f when X changes from -3 to 3 equals .....

(a) zero. (b) 1 (c) -1 (d) $2f(0)$

(36) If f is an odd function and the average rate of change of the function when X changes from -3 to 3 equals 19 , and the average rate of change of the function when X changes from -1 to 1 equals 7 , then the average rate of change of the function when X changes from 1 to 3 equals .....

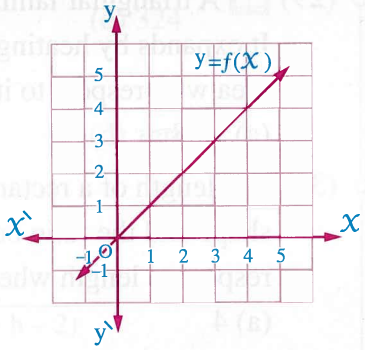
(a) 6 (b) 12 (c) 25 (d) 26

(37) If the average rate of change of the function when x changes from 2 to 3 equals k_1 and the average rate of change of the function when x changes from 3 to 4 equals k_2 , then the average rate of change of the function when x changes from 2 to 4 equals .....

(a) $k_1 + k_2$ (b) $k_1 k_2$ (c) $\frac{1}{2}(k_1 + k_2)$ (d) $2(k_1 + k_2)$

(38) In the given figure :

If the average rate of change of the function f when x changes from 1 to 2 is k_1 and the average rate of change of the function f when x changes from 2 to 4 is k_2 , then .....

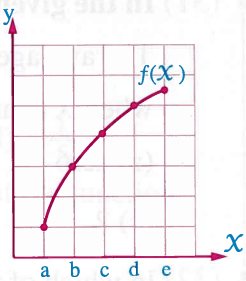


(a) $k_1 = 2 k_2$ (b) $k_1 = \frac{1}{2} k_2$

(c) $k_1 = k_2$ (d) $k_1 = 4 k_2$

(39) The opposite figure shows the curve of the function f where $y = f(x)$

In which of the following the average rate of change in f is the greatest ?



(a) when x changes from a to b (b) when x changes from b to c

(c) when x changes from c to d (d) when x changes from a to e

Second Essay questions

1 **If the function $f : f(x) = x^2 - 3x + 4$, find :**

(1) Variation function when $x = 3$, then calculate the value of V (0.5)

(2) Variation value when x is changed from 2 to 2.4 « 1.75 , 0.56 »

2 **If the function $f : f(x) = x^2 + 2x - 1$, then find the variation in $f(x)$, when :**

(1) x varies from 2 to 2.2 (2) x varies from 2 to 1.8

(3) x varies from 1 to $1 + h$ (4) $x = 2$ and $h = 0.5$

(5) $x = 3$ and $\Delta x = 0.2$ « 1.24 , – 1.16 , $h^2 + 4h$, 3.25 , 1.64 »

3 If $f(x) = x^2 + 3x - 1$, **find :**

(1) The average rate of change function when $x = 2$, then find A (0.2)

(2) The average rate of change when x varies from 4.5 to 3 « 7.2 , 10.5 »

4 **Find the average rate of change of the function $f : f(x) = 2x^2 - 3x + 4$, when x changes from x_1 to $x_1 + h$, then find :**

(1) The average rate of change of f at $x = 3$, then calculate A (0.2)

(2) The average rate of change of f when x changes from 3.5 to 4

(3) The rate of change of f at $x = 2$ « 9.4 , 12 , 5 »

5 **Let the function $f : f(x) = \frac{x+2}{x-2}$, find :**

(1) The average rate of change of f when x varies from x_1 to $x_1 + h$

(2) The average rate of change when x varies from 3 to $3\frac{1}{3}$

(3) The rate of change of f at $x = 4$ « $\frac{-4}{(x_1 - 2)(x_1 + h - 2)}$, -3 , -1 »

6 Find the average rate of change of the function $f : f(x) = \sqrt{x+4}$ when x varies from x_1 to $x_1 + h$, then find this average at $x_1 = 5$ and $h = 1.24$

Find the rate of change of f at $x = 5$ « $\frac{5}{31}$, $\frac{1}{6}$ »

7 Find the average rate of change function in $f(x) = \sqrt{x-5}$ when $x = x_1$, then deduce the rate of change in f when $x = 9$ Can the rate of change in f when $x = 5$ be calculated ? Explain.

8 Let the function $f : f(x) = \tan x$, find the rate of change of f at $x = \pi$ « 1 »

9 If $f(x) = x^5$, find the rate of change of the function f at $x = 2$ « 80 »

10 If $f(x) = a x^2 + b x + 4$, find at $x = 3$ the variation function V (h)

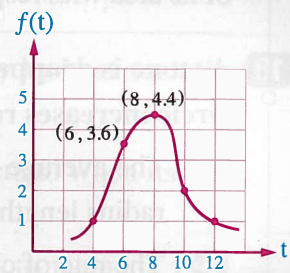
Find the values of a and b where $f(3) = 4$ and $V\left(\frac{1}{2}\right) = 1\frac{3}{4}$ « 1 , -3 »

11 If the function $f : f(x) = 1 + a x + b x^2$ and the change in f when x varies from 3 to 2 equals 7 and the rate of change of f at $x = 3$ equals -9 , then find a and b « 3 , -2 »

12 **The opposite figure shows the curve of r = f (t) where r is the total sales of a computer store approximated in millions L.E. and t is the time months.**
From the graph find the average rate of change in the total sales when the time varies from :

(1) t = 4 to t = 8 | (2) t = 8 to t = 10

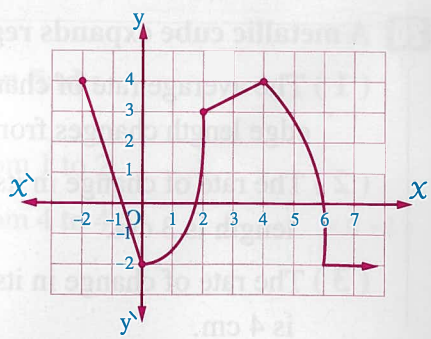
(3) t = 4 to t = 12



« 0.85 , -1.2 , zero »

13 **Creative thinking :**

The opposite figure shows the curve of the function f where $y = f(x)$ Determine the intervals in which the average rate of change in f is constant. Explain « $]-2 , 0[$, $]2 , 4[$, $]6 , \infty[$ »



14 A squared lamina expands regularly keeping its shape. Calculate the average rate of change in its surface area with respect to its side length when its side length varies from 3 cm. to 3.4 cm. , then calculate the rate of change in its surface area with respect to its side length when its side length = 5 cm. « 6.4 , 10 »

15 A squared lamina shrinks by cooling and maintaining its square shape. Calculate the rate of change in the surface area of the lamina with respect to its side length when the side length is 8 cm. « 16 »

16 **A metal lamina in the shape of a rectangle whose length exceeds its width by 3 cm. expands regularly such that it preserves its shape. Find :**

(1) The change in the area of the lamina when its width changes from 4 cm. to 4.2 cm.

(2) The change in the perimeter of the lamina when its width changes from 3.5 cm. to 3.7 cm. « 2.24 , 0.8 »

17 **A lamina in the shape of a rectangle whose length is twice its width expands by heat such that it preserves its shape with the same ratio between its dimensions. Find :**

(1) The average rate of change in its area with respect to its length when its length changes from 15 cm. to 16.5 cm.

(2) The rate of change in each of its area and its perimeter with respect to its length when its length equals 15 cm. « 15.75 , 15 , 3 »

18 A circular disc expands regularly such that it preserves its shape. Find the rate of change of its area with respect to its radius length when the radius length is 14 cm. $\left(\pi = \frac{22}{7}\right)$ « 88 »

19 **A stone is dropped in water to make a circular wave such that the radius of the circle increases regularly. Find :**

(1) The average rate of change in the area of the wave with respect to its radius when its radius length changes from 6 cm. to 6.3 cm.

(2) The rate of change in the area of the wave with respect to its radius when its radius length = 5 cm. « 12.3 π , 10 π »

20 **A metallic cube expands regularly by heat such that it preserves its shape. Find :**

(1) The average rate of change in its total area with respect to its edge length when its edge length changes from 2 cm. to 2.1 cm.

(2) The rate of change in its total area with respect to its edge length when its edge length is 3 cm.

(3) The rate of change in its volume with respect to its edge length when its edge length is 4 cm. « 24.6 , 36 , 48 »

21 A sphere shaped soap bubble expands regularly to maintain its spherical shape. Calculate the average rate of change in its spherical surface area with respect to its radius length when its radius length varies from 0.5 cm. to 0.6 cm. know that the surface area of the sphere equals $4\pi r^2$ where r is the sphere radius length. « 4.4π »

22 If y is the quantity (measured in kg.) which represents the average production of an orange tree depends upon the number of kilograms x of insecticide used to spray the tree according to the relation $y = 100 - \frac{42}{x+1}$, calculate the average rate of change in y with respect to x when x varies from 1 to 2 « 7 »

23 **If the distance S which a moving body covers in a straight line during the time t in seconds is given by : $S = t^2 + 3t + 2$, find :**

(1) The average rate of change in S with respect to time when t changes from 2 to 4 seconds.

(2) The rate of change in S with respect to t when t = 5 seconds. « 9 , 13 »

24 **If the growth of a population follows the relation : $f(t) = 6t^2 + 50000$, where t is measured in days. Find :**

(1) The average rate of growth when t changes from t_1 to $t_1 + h$

(2) The average rate of growth during a time equals 6 days from the start of the third day.

(3) The average rate of growth during the 7^{th} day.

(4) The rate of the instantaneous growth change when t = 4 « $6h + 12t_1$, 60 , 78 , 48 »

25 **A lamina in the shape of an equilateral triangle expands regularly by heat such that it preserves its shape. Find :**

(1) The average rate of change in its area with respect to its side length when its side changes from 3.5 cm. to 4.5 cm.

(2) The rate of change in its area with respect to its side length when its height is $3\sqrt{3}$ cm. « $2\sqrt{3}$, $3\sqrt{3}$ »

26 **If the function f : $f(x) = \begin{cases} 2x^2 - 3 & , x \leq 2 \\ 2x + 1 & , x > 2 \end{cases}$, then find :**

(1) The average rate of change of f when x changes from 1 to 2

(2) The average rate of change of f when x changes from 4 to 4.5 « 6 , 2 »

Exercise

10

Differentiation

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $\lim_{x \to 2^-} f(x) = 2k + 3$, $\lim_{x \to 2^+} f(x) = k - 1$ and the function is differentiable at $x = 2$, then k = ...

(a) 2 (b) – 2 (c) 4 (d) – 4

(2) The slope of the tangent to the curve of the function f at the point (x_1 , y_1) which lies on it equals .....

(a) $f(x_1 + h) - f(x_1)$ (b) $\frac{f(x_1 + h) - f(x_1)}{h}$

(c) $\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$ (d) $\lim_{x \to 0} \frac{f(x_1) - f(h)}{x_1 - h}$

(3) All the following statements are false except .....

(a) If the function is continuous at a point , then it is differentiable at this point.

(b) If the function is not differentiable at a point , then the function is undefined at this point.

(c) If the function f is not continuous at a point , then the function is not differentiable at this point.

(d) If the function has right derivative and left derivative at a point , then it is differentiable at this point.

(4) If f is a function and $f(1) = 5$, $\grave{f}(1) = 4$, then $\lim_{x \to 1} f(x)$ = ...

(a) 5 (b) 4 (c) 9 (d) not exist.

(5) The function $f : f(x) = |x - 5|$ at $x = 5$ .....

(1) is continuous. (2) is differentiable. (3) has a limit.

(a) (1) only. (b) (1) , (2) only. (c) (1) , (3) only. (d) (1) , (2) and (3)

(6) If $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ x^2 - 3 & , \quad x < 1 \end{cases}$, then $\grave{f}(1^+)$ = ...

(a) 2 (b) -2 (c) 4 (d) not exist.

(7) If $f(x) = \begin{cases} x^2 & , \quad x \leq 2 \\ 4x & , \quad x > 2 \end{cases}$, then $\grave{f}(2)$ = ...

(a) 2 (b) 4 (c) 8 (d) not exist.

(8) If $f : f(x) = \begin{cases} x^2 + a & , \quad x \leq 2 \\ a x + b & , \quad x > 2 \end{cases}$ is differentiable at $x = 2$, then $a + b$ = ...

(a) 4 (b) -4 (c) -8 (d) 8

(9) If $f : f(x) = \begin{cases} a x + 5 b & , \quad x > 2 \\ 26 & , \quad x = 2 \\ b x^2 + 3 a - 6 & , \quad x < 2 \end{cases}$, is differentiable at $x = 2$, then $a - b$ = ...

(a) zero (b) 4 (c) 6 (d) 8

(10) If $f(x) = \begin{cases} x^2 + x + 1 & , \quad x \geq 1 \\ 3x & , \quad x < 1 \end{cases}$, then the function f at $x = 1$ .....

(a) is continuous but not differentiable. (b) is differentiable.

(c) has a vertical tangent. (d) has a horizontal tangent.

(11) If the function f is continuous at $x = 3$ and $\lim\limits_{x \to 3} \dfrac{2x - 6}{f(x) - f(3)} = \dfrac{1}{6}$, then $\grave{f}(3)$ = ...

(a) -12 (b) 10 (c) 12 (d) -10

(12) When x changes from x_1 to x_2 , which of the following has a greater average rate of change ?

(a) $f(x) = 4x + 10$ (b) $f(x) = 6x - 3$ (c) $f(x) = \frac{1}{2}x + 2$ (d) $f(x) = 2x - 1$

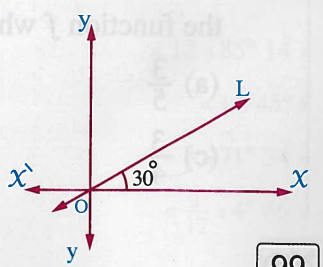
(13) If f is a function where $f(2 + h) = f(2) + h$, then $\grave{f}(2)$ = ...

(a) zero (b) 1 (c) 2 (d) undefined.

(14) If the curve of the function f in the given figure is represented by the straight line L , then the average rate of change of the function f is .....

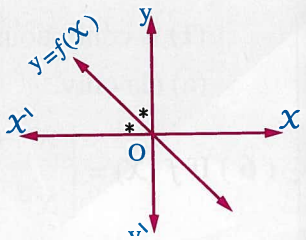
(a) $\tan 30°$ (b) $\sin 30°$

(c) $\cos 30°$ (d) $-\tan 30°$



(15) In the opposite figure :

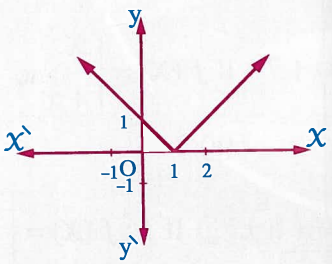
The average rate of change of the function f equals .....



(a) 1 (b) – 1

(c) X (d) $-X$

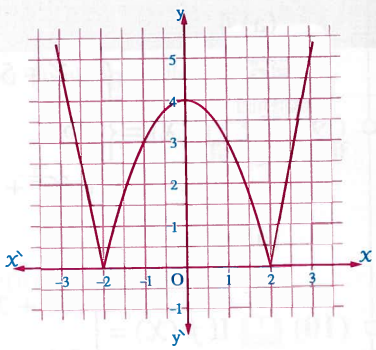
(16) The opposite figure represents the curve of function f , then $\grave{f}\ (1^-)$ = ...



(a) 1 (b) zero

(c) – 1 (d) undefined.

(17) The opposite figure represents the curve of the function f , then the value of X at which the function is not differentiable is .....



(a) – 2 only

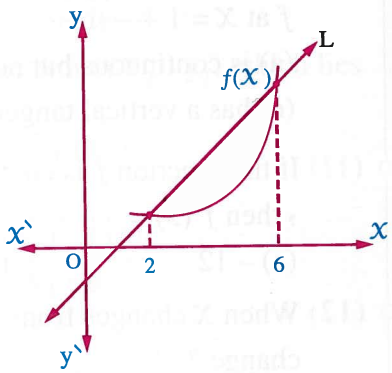
(b) 2 only

(c) 4 only

(d) (a) and (b) together.

(18) In the opposite figure :

If the slope of the line L equals 3 , then $f\ (6) - f\ (2)$ = ...



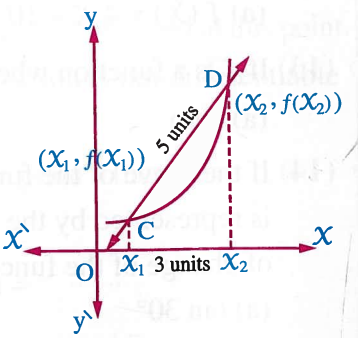
(a) 4

(b) 7

(c) 12

(d) 24

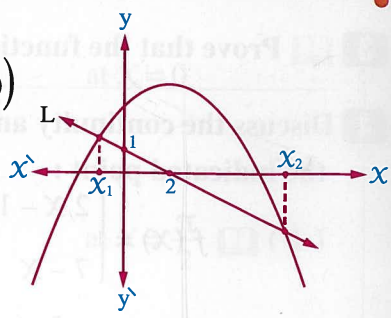
(19) The given figure represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $y = f\ (X)$, $\overleftrightarrow{CD}$ intersects the curve at C and D , then the average rate of change of the function f when X changes from X_1 to X_2 = ...



(a) $\frac{3}{5}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

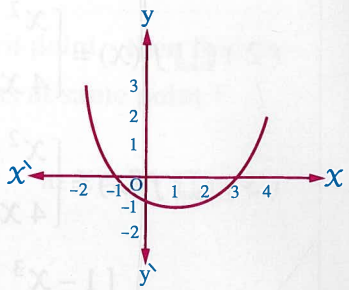
(20) The given figure represents the curve of a function f
If the line L intersects the curve at the points $(x_1 , f(x_1))$, $(x_2 , f(x_2))$, then the average rate of change of f when x varies from x_1 to x_2 equals .....



(a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

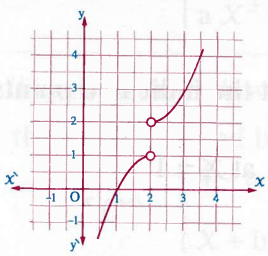
(21) The opposite figure represents the curve of the function f , then the function $g : g(x) = |f(x)|$ is not differentiable at $x \in$ .....

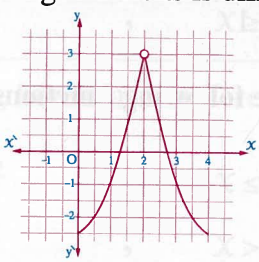


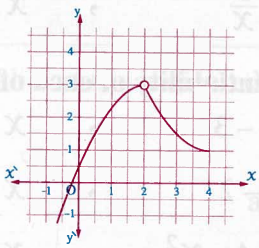
(a) $\{3\}$ (b) $\{3 , -1\}$

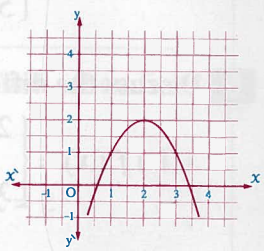
(c) $\{1\}$ (d) $\varnothing$

(22) Which of the following functions is differentiable at $x = 2$?


(a)


(b)


(c)


(d)

Second Essay questions

1 **Using the definition of the derivative , find $\dot{f}(x)$ for the following functions :**

(1) $f(x) = 8$

(2) $f(x) = 5x - 7$

(3) $f(x) = x^2 + 5$

(4) $f(x) = 9x + x^2$

(5) $f(x) = x^3 - 2$

(6) $f(x) = \frac{1}{x}$

(7) $f(x) = \frac{7}{x} - 2$

(8) $f(x) = 5 - 4\sqrt{x}$

(9) $f(x) = \sqrt{3x + 1}$

(10) $f(x) = \frac{x}{x + 5}$

2 **Using the definition of the derivative , find the slope of the tangent to each curve of the following functions at the indicated point , then find the measure of the positive angle which this tangent makes with the positive direction of x-axis at the same point to the nearest minute :**

(1) $f(x) = 3x^2 - 5$ at the point $(2 , 7)$ « 12 , 85° 14̂ »

(2) $f(x) = 1 - 5x - 3x^2$ at the point $(-1 , 3)$ « 1 , 45° »

(3) $f(x) = x^3 - 4$ at the point $(1 , -3)$ « 3 , 71° 34̂ »

(4) $f(x) = \sqrt[3]{x + 3}$ at the point $(5 , 2)$ « $\frac{1}{12}$, 4° 46̂ »

3 Prove that the function $f : f(x) = x^2 - x + 1$ is differentiable at $x = 1$

4 Discuss the continuity and the differentiability of each of the following functions at the indicated point :

(1) $f(x) = \begin{cases} 2x - 1 & , \quad x < 3 \\ 7 - x & , \quad x \geq 3 \end{cases}$ at $x = 3$

(2) $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ 4x - 1 & , \quad x \leq 2 \end{cases}$ at $x = 2$

(3) $f(x) = \begin{cases} x^2 - 5 & , \quad x < 2 \\ 4x - 9 & , \quad x \geq 2 \end{cases}$ at $x = 2$

(4) $f(x) = \begin{cases} 1 - x^3 & , \quad x > -1 \\ 5 + \frac{3}{x} & , \quad x \leq -1 \end{cases}$ at $x = -1$

5 Discuss the differentiability of each of the following functions at the indicated points :

(1) $f(x) = \begin{cases} 2x - 3 & , \quad x \leq 1 \\ x^2 - 2 & , \quad x > 1 \end{cases}$ at $x = 1$

(2) $f(x) = \begin{cases} 4 - x^2 & , \quad x \leq 1 \\ 2x + 1 & , \quad x > 1 \end{cases}$ at $x = 1$

(3) $f(x) = \begin{cases} x^2 + 4x - 3 & , \quad x < 1 \\ 3x^2 - 12x + 11 & , \quad x \geq 1 \end{cases}$ at $x = 1$

6 Show that the function f where $f(x) = \begin{cases} x^2 & \text{when } x \leq 2 \\ x + 2 & \text{when } x > 2 \end{cases}$ is not-differentiable when $x = 2$

7 Discuss the differentiability of each of the following functions at the indicated points :

(1) $f(x) = \frac{x - 1}{x + 1}$ at $x = 2$

(2) $f(x) = x + \frac{1}{x}$ at $x = 1$

(3) $f(x) = \sqrt{x - 4}$ at $x = 4$

(4) $f(x) = \sqrt[3]{(x + 2)^2}$ at $x = 6$

(5) $f(x) = |x - 2|$ at $x = 2$

(6) $f(x) = \sqrt{x^2 + 6x + 9}$ at $x = -3$

(7) $f(x) = 3 + |x + 2|$ at $x = -2$

(8) $f(x) = x|x|$ at $x = 0$

8 Discuss the differentiability of each of the following functions at the indicated points :

(1) $f(x) = \begin{cases} x|x| + 5 & , \quad x \geq 0 \\ \frac{x}{|x|} + 6 & , \quad x < 0 \end{cases}$ at $x = 0$

(2) $f(x)=\begin{cases}2x+3 & , \quad x\le 0\\ 3+\tan 2x & , \quad x>0\end{cases}$ at $x=0$

(3) $f(x)=\begin{cases}\cos 2x & , \quad x\le \frac{\pi}{2}\\ -\sin x & , \quad x>\frac{\pi}{2}\end{cases}$ at $x=\frac{\pi}{2}$

9 **If each of the following functions is continuous at the indicated point , then find the value of a , then discuss the differentiability of these functions at same point :**

(1) $f(x)=\begin{cases}a x^2+1 & , \quad x\ge 2\\ 4x-3 & , \quad x<2\end{cases}$ at $x=2$ « 1 »

(2) $f(x)=\begin{cases}3x+2 & , \quad x<2\\ a x^3 & , \quad x\ge 2\end{cases}$ at $x=2$ « 1 »

10 **If each of the following functions is differentiable at the indicated point , then find the value of a and b :**

(1) $f(x)=\begin{cases}4-a x^2 & , \quad x\le 2\\ x+b & , \quad x>2\end{cases}$ at $x=2$ « $\frac{-1}{4}$, 3 »

(2) $f(x)=\begin{cases}x^2-4 & , \quad x\le 2\\ a x+b & , \quad x>2\end{cases}$ at $x=2$ « 4 , – 8 »

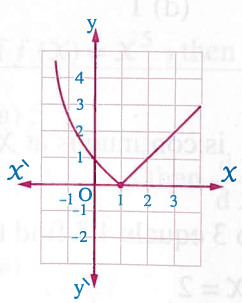
11 **If $f(x)=a x^2+b$ where a and b are two constants , find :**

(1) The first derivative of the function f at any point (x , y)

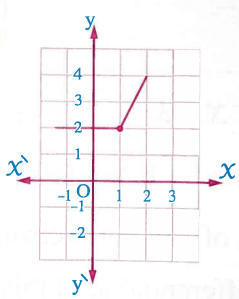
(2) The two values of a and b if the slope of the tangent to the curve of the function at the point $(2 , -3)$ lying on it equals 12 « 3 , – 15 »

12 **Compare the right derivative and the left derivative for each of the following functions and prove that each of them is not-differentiable at point $x=1$**

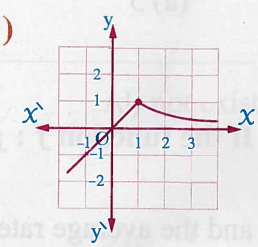
(1)



(2)



(3)



13 If $f(x) = \begin{cases} x^2 + a x - 2 & , \quad x < 2 \\ 4 & , \quad x = 2 \\ a + b x & , \quad x > 2 \end{cases}$ is continuous at $x = 2$

, then find the values of a and b , then discuss the differentiable of f at $x = 2$ « 1 , 1.5 »

---

14 $f(x) = \begin{cases} m x + 3 c & , \quad x < 1 \\ c x^2 + 4 m & , \quad x \geq 1 \end{cases}$ is continuous at $x = 1$

, $f(1) = 11$, find the values of m and c , then discuss the differentiable of the function at $x = 1$ « 2 , 3 »

Third Higher skills

1 **Choose the correct answer from the given ones :**

(1) If $f(a) =$ zero , $\grave{f}(a) = b$ where $b > 0$ and $g(x) = |f(x)|$, then $\grave{g}(a) =$ .....

(a) b (b) – b (c) zero (d) undefined.

(2) $\lim\limits_{h \to 0} \dfrac{f(h+3) - f(h-2) + f(-2) - f(3)}{h} =$ .....

(a) $\grave{f}(3) + \grave{f}(-2)$ (b) $\grave{f}(3)$

(c) $\grave{f}(-2)$ (d) $\grave{f}(3) - \grave{f}(-2)$

(3) If f is a function , $\grave{f}(1) = 2\, f(1) = 4$, then $\lim\limits_{h \to 0} \dfrac{3h}{f(1+h) - 2} =$ .....

(a) zero (b) $\frac{3}{4}$ (c) 12 (d) $\frac{1}{4}$

(4) If f is a function and $f(2) = 4$, $\grave{f}(2) = 1$, then $\lim\limits_{x \to 2} \dfrac{x f(2) - 2 f(x)}{x - 2} =$ .....

(a) zero (b) 1 (c) 2 (d) 4

(5) If the curve of the function $f : f(x) = x^2 - 1$ passes through the points A $(2 , f(2))$, B $(3 , f(3))$, then $\dfrac{\text{the slope of the secant } \overleftrightarrow{AB}}{\text{the slope of the tangent to the curve at A}} =$ .....

(a) 5 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 1

---

2 **If the function f :** $f(x) = \begin{cases} 2x + 3 & , \quad x < 2 \\ a x^2 + b x - 1 & , \quad x \geq 2 \end{cases}$ is continuous at $x = 2$

and the average rate of change of f when x changes from 1 to 3 equals 4.5 find the value of a and b , then discuss the differentiable of this function at $x = 2$ « 1 , 2 »

Exercise

11

Rules of differentiation

From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $f(x) = \frac{1}{x}$, then $f'(1) =$ .....

(a) 1 (b) zero (c) -1 (d) 2

(2) $\frac{d}{dx}[(x-1)(x+3)] =$ .....

(a) $2x$ (b) $2x+1$ (c) $2x-3$ (d) $2x+2$

(3) If $f(x) = \frac{1}{2}x^{14} - \frac{1}{4}x^{20} + \frac{1}{6}x^{36}$, then $f'(1) =$ .....

(a) 6 (b) 8 (c) 16 (d) 18

(4) If $f(x) = (4x^2 - 1)(1 + 4x^2)$, then $f'\left(-\frac{1}{2}\right) =$ .....

(a) zero (b) -1 (c) -8 (d) -64

(5) If $f'(2) = 5$, $f(x) = x^2 + mx + 5$, then $m =$ .....

(a) 5 (b) 2 (c) 10 (d) 1

(6) If $f(x) = x^5$, then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$ .....

(a) 5 (b) $5x^4$ (c) x^5 (d) not exist.

(7) If $y = \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx} =$ .....

(a) $\frac{-1}{2\sqrt[3]{x^2}}$ (b) $\frac{-1}{2x}$ (c) $\frac{-1}{2x\sqrt{x}}$ (d) $\frac{1}{2\sqrt{x}}$

(8) If $f(x) = a^2$ where a is a constant, then $f'(x) =$ .....

(a) zero (b) $2a$ (c) 2 (d) a^2

(9) If $y = 3\sqrt[4]{x}$, then $\frac{dy}{dx} =$ .....

(a) $12x^3$ (b) $12\sqrt[3]{x}$ (c) $\frac{3}{4\sqrt[3]{x^4}}$ (d) $\frac{3}{4\sqrt[4]{x^3}}$

(10) If $y = \frac{2}{x} + \frac{x}{2} + 2$, then $\frac{dy}{dx} =$ .....

(a) $\frac{2}{x^2} + \frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{x^2-4}{2x^2}$ (d) $\frac{x^2-2}{2x^2}$

(11) If $y = \frac{x^2+6}{x^3+5}$, then $\dot{y} =$ ..... at $x = 1$

(a) $\frac{-1}{4}$ (b) $\frac{6}{5}$ (c) $\frac{7}{6}$ (d) $\frac{11}{12}$

(12) $\frac{d}{dx}\left(f(x) \cdot g(x)\right) =$ .....

(a) $f'(x) \cdot \dot{g}(x)$ (b) $f(x) \cdot \dot{g}(x)$

(c) $f'(x) \cdot g(x)$ (d) $f(x) \cdot \dot{g}(x) + f'(x) \cdot g(x)$

(13) If $y = (x+4)(x^2+5x+7)$, then $\dot{y} =$ .....

(a) $2x^2 + 13x + 20$ (b) $x^2 + 18x + 20$

(c) $x^3 + 18x^2 + 27x + 28$ (d) $3x^2 + 18x + 27$

(14) If $xy = 81$, then $\frac{dy}{dx} =$ ..... at $x = 9$

(a) -9 (b) -1 (c) 1 (d) 9

(15) If $f(x) = x^n + x^{-n}$, then $\dot{f}(1) =$ .....

(a) n (b) $n + \frac{1}{n}$ (c) $n - \frac{1}{n}$ (d) zero

(16) If $n \in \mathbb{Z}^+$ and $f(x) = nx^n$, $\dot{f}(1) = 9$, then $n =$ .....

(a) 2 (b) 3 (c) 4 (d) 5

(17) $\frac{d}{dx}(5\pi^2) =$ .....

(a) 5 (b) 10π (c) zero (d) $5\pi^2$

(18) If $f(x) = x + \frac{9}{x}$, then $\dot{f}(x) =$ zero at $x =$ .....

(a) 3 (b) -3 (c) ± 3 (d) ± 9

(19) If $f(x) = 4x^2 + g(x)$ and $\dot{f}(3) = 7$, then $\dot{g}(3) =$ .....

(a) 24 (b) 31 (c) -17 (d) -24

(20) If $f(x) = 5\,g(x) + 20$, then $\grave{g}(x) =$ .....

(a) $\grave{f}(x)$ (b) $\grave{f}(x) - 20$ (c) $5\,\grave{f}(x)$ (d) $\frac{1}{5}\,\grave{f}(x)$

(21) If $f(x) = (x + a)(x^2 - a)$ and $\grave{f}(-1) = 4$, then a = ...

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

(22) If $f(x) = m\,x^2 + n\,x + \ell$, then $\grave{f}(1) + \grave{f}(4) - \grave{f}(5) =$ .....

(a) m (b) – m (c) n (d) – n

(23) If $f(x) = \frac{1}{3}\,x^3 - 2\,x^2 + 5\,x - 4$ and $\grave{f}(x) = 2$, then $x =$ .....

(a) 1 or 2 (b) 1 or 3 (c) 2 or 3 (d) 3 or 4

(24) If $f(x) = a\,x^2$, $g(x) = \frac{x^3}{b}$, $\grave{f}(-3) = \grave{g}(2)$, then a b = ...

(a) – 6 (b) – 2 (c) 2 (d) 12

(25) If $f(x) = 2\,a\,x^3$, $\grave{f}(-3) = 108$, then $\grave{f}(1) =$ .....

(a) 2 (b) 6 (c) 12 (d) 24

(26) If $y = \sqrt[n]{x^m}$, then $\frac{dy}{dx} =$ .....

(a) $\frac{m\,y}{n\,x}$ (b) $m\sqrt[n]{x^{m-1}}$ (c) $\frac{m}{n}\sqrt[n]{x^{m-1}}$ (d) $\frac{m}{n}\sqrt[n]{x^{m+n}}$

(27) If $y = \frac{x^2 + 2x + 4}{x^2 - 2x + 4}$ and $\frac{dy}{dx} =$ zero , then $x =$ .....

(a) ± 2 (b) ± 4 (c) 2 or 3 (d) – 2 or – 3

(28) If $f(x) = \frac{2x^2 + a\,x + b}{x^2 - 5x + 4}$ and $f(0) = 3$, $\grave{f}(0) =$ zero , then a × b = ...

(a) 180 (b) – 180 (c) 3 (d) – 27

(29) If $f(x) = x^3 - a\,x + 1$, $g(x) = a\,x^2 + b\,x + 3$

and $2\,\grave{f}(1) + \grave{g}(1) = 2$, then b = ...

(a) – 4 (b) – 3 (c) 3 (d) 4

(30) If $f(x) = x^2 + a\,x + b$ and $\grave{f}(5) = 2\,\grave{f}\left(\frac{7}{2}\right)$, then a = ...

(a) – 4 (b) – 3 (c) 2 (d) 3

(31) If $(f \square g)\grave{} = \grave{f} \square \grave{g}$, then which of the following operations could be placed in the square to make a true statement for all differentiable functions f , g

(a) (+) only. (b) (+) , (–) only.

(c) (+) , (–) , (×) , (÷) (d) (×) only.

(32) If $f : [-1, 1] \longrightarrow \mathbb{R}$, where $f(x) = x^2$, $g : [0, 4] \longrightarrow \mathbb{R}$, where $g(x) = 4x$, then $\frac{d}{dx}\left(f(x) \cdot g(x)\right) =$ ..... at $x = 2$

(a) 16 (b) 32 (c) 48 (d) undefined

(33) If $\frac{d}{dx}[(ax)^3] = 24$ at $x = 1$, then the value of a equals .....

(a) 8 (b) 2 (c) 1 (d) 24

(34) If $f(x) = x^3 - 3x^2 - 5x + 6$, then $\lim_{h \to 0} \frac{f(1+2h) - f(1)}{h} =$ .....

(a) -8 (b) 8 (c) 16 (d) -16

(35) If $f(x) = (x-4)(x-5)(x-6)(x-7)$, then $\grave{f}(7) =$ .....

(a) -7 (b) -6 (c) zero (d) 6

(36) If $f(x) = x^2 - x + 4$, and $\grave{f}(k) = \grave{f}(2) + f(2)$, then k = ...

(a) 7 (b) 5 (c) 3 (d) 1

(37) If $f : f(x)$ is an even differentiable function for all values of $x \in \mathbb{R}$, then $\grave{f}(a) + \grave{f}(-a) =$ .....

(a) zero (b) $2\grave{f}(a)$ (c) $2\grave{f}(-a)$ (d) $-2\grave{f}(a)$

(38) If the function f is an odd function differentiable for each $x \in \mathbb{R}$, then $\grave{f}(a) + \grave{f}(-a) =$ .....

(a) zero (b) $2\grave{f}(a)$ (c) 1 (d) $2f(-2)$

(39) If $f : f(x) = ax^2 - bx + 1$, $\grave{f}(2) = 2$, the average rate of change of the function when x changes from 1 to 1.2 equals 5.6, then a + b = ...

(a) -10 (b) -2 (c) 12 (d) -12

(40) If $f(x) = \begin{vmatrix} x & -1 \\ 1 & 2x \end{vmatrix}$, then $\grave{f}(2) =$ .....

(a) 10 (b) 8 (c) 6 (d) 2

(41) Which of the following doesn't equivalent to first derivative of the function $f : f(x) = \sqrt{x}$?

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sqrt{x}}{2x}$ (c) $\frac{x}{2\sqrt{x}}$ (d) $\frac{1}{2}x^{-\frac{1}{2}}$

(42) If $f(x) = 3x^2 - 5x + 1$ and $f(a) = \grave{f}(a)$, then a = ...

(a) 3 or $\frac{2}{3}$ (b) $\frac{2}{3}$ or $\frac{1}{3}$ (c) 1 or 3 (d) 2 or 3

(43) If $y = (a\mathcal{X} + 2)^2$ and $\frac{dy}{d\mathcal{X}} = 6$ at $\mathcal{X} = 1$, then a = ...

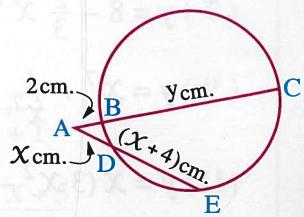
(a) 1 or 6 (b) 1 or – 3 (c) – 3 or 2 (d) 2 or 6

(44) A cuboid with a square base of side length $\mathcal{X}$ cm. and its height y cm. If the volume of the cuboid is 80 cm^3 , then $\frac{dy}{d\mathcal{X}}$ = ... at $\mathcal{X} = 4$

(a) 4 (b) $\frac{-5}{4}$ (c) $\frac{-5}{2}$ (d) $\frac{-2}{5}$

(45) In the opposite figure :

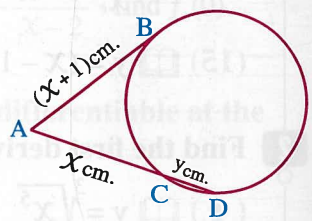
If $\overrightarrow{AC}$ intersects the circle at B , C , $\overrightarrow{AE}$ intersects the circle at D , E , then $\frac{dy}{d\mathcal{X}}$ = ... at $\mathcal{X} = 3$ cm.



(a) 3 (b) 6

(c) 8 (d) 12

(46) If $\overline{AB}$ is a tangent segment of the circle , then $\frac{dy}{d\mathcal{X}}$ = ... when $\mathcal{X} = 5$

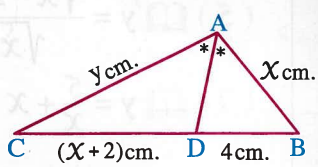


(a) – 5 (b) $-\frac{1}{5}$

(c) $\frac{-1}{25}$ (d) – 25

(47) In the opposite figure :

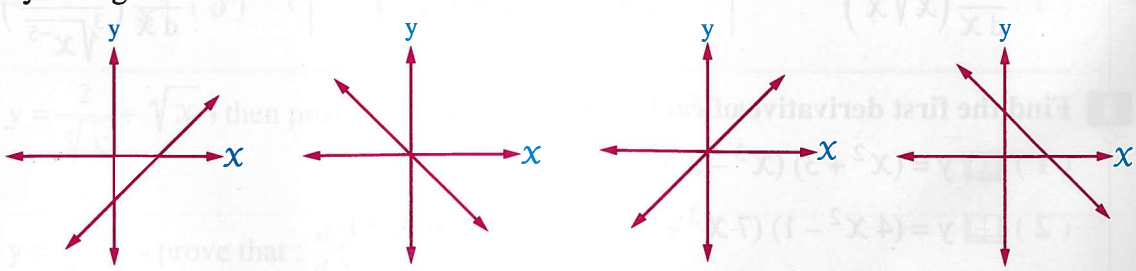
If $\overrightarrow{AD}$ bisects $\angle BAC$, then $\frac{dy}{d\mathcal{X}}$ = ... when $\mathcal{X} = 2$



(a) $\frac{1}{2}$ (b) 1

(c) $1\frac{1}{2}$ (d) 2

(48) If $y = a\mathcal{X}^2 + 7$ is a polynomial function where $a \in \mathbb{R}^-$, then $\frac{dy}{d\mathcal{X}}$ could be represented by the figure :



(a) (b) (c) (d)

Second Essay questions

1 **Find the first derivative of each of the following :**

(1) $y = 7$

(2) $y = 5x$

(3) $y = x^4$

(4) $y = \frac{4}{3}\pi x^3$

(5) $y = \frac{-4}{x^5}$

(6) $f(x) = 3x^2 - 4x + 5$

(7) $y = 8 - \frac{2}{3}x^3 + \frac{1}{5}x^5$

(8) $y = 2x^6 + x^{-9}$

(9) $y = x^7 + \frac{1}{x^7}$

(10) $y = x^4 - 3x^2 + 5 - \frac{2}{x^2} - \frac{3}{x^4}$

(11) $y = x(3x^2 - 2x) + 12x$

(12) $y = \frac{5x^5 - 2x^3 + 10x^2 - 20}{10x^3}$

(13) $y = \frac{2}{x^2}(x^4 - 4x^2 + x - 3)$

(14) $y = \frac{(x-2)(x+2)(x^2+4)}{x^2}$

(15) $y = (x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)(x^{16}+1)$

2 **Find the first derivative of each of the following :**

(1) $y = \sqrt[3]{x^5}$

(2) $y = 2x^6 + 3\sqrt{x}$

(3) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

(4) $y = x(3x^2 - \sqrt{x})$

(5) $y = \frac{\sqrt{x} - 2x}{\sqrt{x}}$

(6) $y = \frac{4x^2 - 3x + 2\sqrt{x}}{x}$

(7) $y = \frac{5}{x} + x\sqrt{x} + \sqrt{3}x - 4$

(8) $y = \sqrt{x}\left(x\sqrt{x} - \frac{x}{\sqrt{x}} - 3\right)$

3 **Find the value of each of the following :**

(1) $\frac{d}{dx}\left(\frac{1}{x^3}\right)$

(2) $\frac{d}{dx}(x^{\frac{1}{3}})$

(3) $\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x}}\right)$

(4) $\frac{d}{dx}\left(x\sqrt{x}\right)$

(5) $\frac{d}{dx}\left(\sqrt[3]{x^2}\right)$

(6) $\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x^{-5}}}\right)$

4 **Find the first derivative of each of the following :**

(1) $y = (x^2 + 3)(x^3 - 3x + 1)$

(2) $y = (4x^2 - 1)(7x^3 + x)$ at $x = 1$ « 130 »

(3) $y = (x^2 + 1)(x^3 + 3)$ at $x = -1$ « 2 »

(4) $f(x) = (4x^2 - 3x + 7)(x^2 + 2x + 3)$ at $(0, 21)$ « 5 »

(5) $f(x) = x(3x - 4)(x^2 - 2)$ at $x = 0$ « 8 »

(6) $y = (2x + 1)(3x^2 - 2)(4x^3 + 3)$

5 **Find the first derivative of each of the following :**

(1) $y = \frac{4}{x^2}$

(2) $y = \frac{5}{2x + 3}$

(3) $f(x) = \frac{x}{2x + 1}$

(4) 📖 $y = \frac{5x - 2}{5x + 1}$

(5) $f(x) = \frac{x}{x^2 - 1}$

(6) 📖 $f(x) = \frac{x^2 - 1}{x^3 + 1}$

(7) $y = \frac{x^2 - 6x + 7}{x - 2}$

(8) 📖 $y = \frac{x^2 + 2x + 5}{x^2 - 5x + 1}$

(9) $f(x) = 1 - \frac{1}{x - 1}$, then find $\hat{f}(3)$ « $\frac{1}{4}$ »

(10) $f(x) = \frac{x}{x + 2} - \frac{x - 1}{x - 2}$, find $\hat{f}(0)$ « $\frac{3}{4}$ »

6 **Find the values of a and b if each of the following functions is differentiable at the indicated point :**

(1) $f(x) = \begin{cases} x^3 + 4 & , x \le 1 \\ ax + b & , x > 1 \end{cases}$ at $x = 1$ « 3 , 2 »

(2) $f(x) = \begin{cases} x^2 - a & , x \le 2 \\ ax + b & , x > 2 \end{cases}$ at $x = 2$ « 4 , – 8 »

(3) $f(x) = \begin{cases} ax^2 + bx + 7 & , x \le 2 \\ \frac{5}{x - 1} & , x > 2 \end{cases}$ at $x = 2$ « – 2 , 3 »

7 📖 If $y = \frac{x^3 - 1}{x^3}$, then prove that : $x^4 \left(\frac{dy}{dx} \right) = 3$

8 If $y = \frac{2}{\sqrt{x}} + \sqrt{x}$, then prove that : $2x^2 y \left(\frac{dy}{dx} \right) = x^2 - 4$

9 If $y = \frac{3}{x - 1}$, prove that : $\frac{dy}{dx} = -\frac{1}{3} y^2$

10 If $y = \frac{\ell^2}{x + \ell^2}$, ℓ is constant , prove that : $\frac{dy}{dx} + \left(\frac{y}{\ell}\right)^2 = 0$

11 If $f(x) = a x^3 + b x$, $f(1) = 2$, $f'(1) =$ zero , find a , b « – 1 , 3 »

12 Find the values of x which make $f'(x) = 7$,
where $f(x) = x^3 - 5x + 2$ « ± 2 »

13 Find the value of a if : $f(x) = \frac{x + a}{x - a}$ and $f'(2) = -2$ « 4 or 1 »

14 If $f(x) = \frac{x^2 + a x + b}{x^2 - x + 2}$, $f(0) = 1$ and $f'(0) = 1$, find a , b « 1 , 2 »

15 If $y = a x^3 + b x^2$ and $\frac{dy}{dx} = 8$ when $x = 1$, and the average rate of change of y when x varies from –1 to 2 equals 7 , find the values of the two constants a and b « 2 , 1 »

Third Higher skills

Choose the correct answer from those given :

(1) If $\dot{f}(x) \times g(x) + \dot{g}(x) \times f(x) = x + \frac{1}{x}$, then $\frac{d}{dx}[f(x) \times g(x)] =$ ..... at $x = 2$

(a) $\frac{3}{4}$ (b) 1 (c) $\frac{5}{2}$ (d) 3

(2) If $f(x) = (2x + 1) \times g(x)$ and $f(2) = 15$, $\dot{g}(2) = 4$, then $\dot{f}(2) =$ .....

(a) 26 (b) 28 (c) 30 (d) 32

(3) If $2 f(1) = \dot{f}(1) = 4$, $3 g(1) = \dot{g}(1) = 3$, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ ..... at $x = 1$

(a) – 4 (b) – 2 (c) 1 (d) 3

(4) If $f(x) = \frac{g(x)}{2x^2 + 3}$ and $g(1) = -5$, $\dot{g}(1) = 6$, then $\dot{f}(1) =$ .....

(a) 1 (b) 2 (c) 3 (d) 4

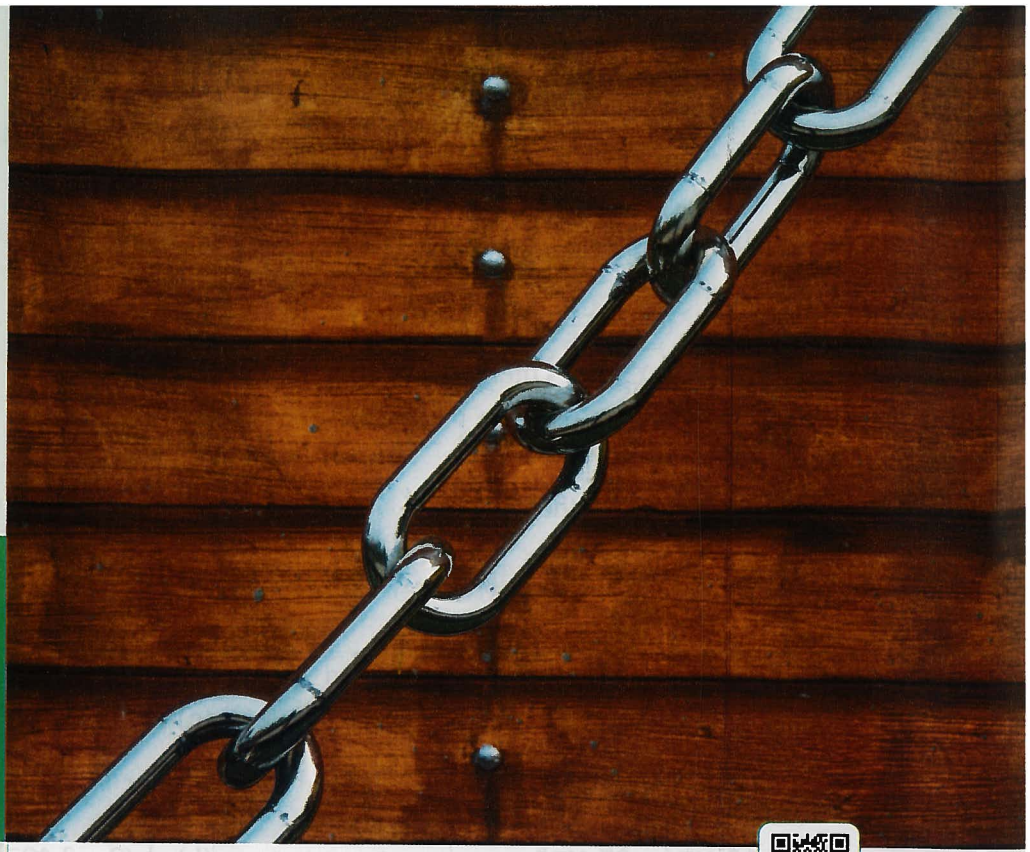
(5) If $f(x) = (x - 1)(x - 2)(x - 3)$, then $\sum_{k=1}^{3} \dot{f}(k) =$ .....

(a) 1 (b) 2 (c) 3 (d) 4

Exercise

12

Derivative of composite function (Chain rule)



From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from those given :

(1) If $y = x^2$, $x = z^3 - 1$, then $\frac{dy}{dz}$ = ...

(a) $z^3 - 1$ (b) $6z^2$ (c) $6z^2(z^3 - 1)$ (d) $x^2(z^3 - 1)$

(2) If $y = x^2 + x$, $x = z^2 + z$, then $\frac{dy}{dz}$ = ... at $z = -1$

(a) -1 (b) 1 (c) zero (d) 2

(3) If $y = z^3 - z - 1$, $z = x - \frac{4}{x}$, then $\frac{dy}{dx}$ = ... at $x = 2$

(a) -1 (b) -2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(4) If $y\,x = 5$, $x\,z = 3$, then $\frac{dy}{dz}$ = ...

(a) 15 (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) zero

(5) If $y = \frac{x-5}{x+5}$, $x = 2z + 3$, then $\frac{dy}{dz}$ = ... at $z = 1$

(a) $\frac{1}{25}$ (b) $-\frac{1}{25}$ (c) $-\frac{1}{5}$ (d) $\frac{1}{5}$

(6) If $z = \frac{1}{2}y^2 + y - 2$, $y = x^2 + x + 1$, then $\frac{dz}{dx}$ = ... at $x = -\frac{1}{2}$

(a) zero (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(7) If $y = \sqrt{z} + \frac{1}{\sqrt{z}}$, $z = x^2 + 25$, then $\frac{dy}{dx}$ = ... at $x = 0$

(a) -2 (b) zero (c) 1 (d) 3

(8) If $y = \frac{z+3}{z-1}$, $z = \frac{x+1}{x-3}$, then $\frac{dy}{dx}$ = ... at $x = 4$

(a) zero (b) 1 (c) 2 (d) 3

(9) If $f(x) = \sqrt{x^2+9}$, then $\dot{f}(-4)$ = ...

(a) $\frac{-4}{5}$ (b) 5 (c) $\frac{1}{10}$ (d) $\frac{-1}{10}$

(10) If $y = (x^2-2)^5$, then $\frac{dy}{dx}$ = ...

(a) $5(x^2-2)^4$ (b) $10x(x^2-2)^4$ (c) $22x^5$ (d) $2x(x^2-2)^5$

(11) If $y = (z+1)^3$, $z = x^5 - 1$, then $\frac{dy}{dx}$ = ...

(a) x^{15} (b) x^8 (c) $15x^{14}$ (d) $8x^7$

(12) $\frac{d}{dx}(2-3x)^{-2}$ = ...

(a) $12x^{-2} - 27x^{-4}$ (b) $\frac{1}{3}(2-3x)^{-3}$

(c) $6(2-3x)^{-3}$ (d) $-2(2-3x)^{-3}$

(13) If $y = \sqrt{2-x}$, then $\frac{dy}{dx}$ = ...

(a) $-\frac{1}{2y}$ (b) -1 (c) $-2y-1$ (d) $-2y$

(14) If $y = (x+k)^3$, $\frac{dy}{dx} = 12$ at $x = 0$, then k = ...

(a) 2 only. (b) ± 2 (c) -2 only. (d) 4 only.

(15) If $y = \frac{1}{\sqrt{x^2+1}}$, then $\dot{y}$ = ...

(a) $\frac{-x}{\sqrt{x^2+1}}$ (b) $\frac{-x}{\sqrt[3]{x^2+1}}$ (c) $\frac{-x}{\sqrt{(x^2+1)^3}}$ (d) $\frac{-x}{\sqrt[3]{(x^2+1)^2}}$

(16) If $y = \sqrt[3]{2x^2-3x+9}$, then $\frac{dy}{dx}$ = ... at $x = 1$

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) 6 (d) 12

(17) If $y = f(x)$, then $\frac{d}{dx}(y^6)$ = ...

(a) $6y^5$ (b) y^6 (c) zero (d) $6y^5\frac{dy}{dx}$

(18) If $y^4 = x^3$, then $\frac{dy}{dx}$ = ...

(a) $\frac{3x}{4y}$ (b) $\frac{x^2}{4y^3}$ (c) $\frac{3x^2}{y^3}$ (d) $\frac{3y}{4x}$

(19) If $y = \frac{z}{z-1}$, $z = \frac{x}{x+1}$, then .....

(a) $\frac{dy}{dx} + 1 = 0$ (b) $\frac{dy}{dz} = \frac{dz}{dx} \times \frac{dx}{dy}$

(c) $\frac{dy}{dx} = 1$ (d) $\frac{dy}{dx} = \frac{z\,x}{(z-1)(x+1)}$

(20) If $x^2 + y^2 = y$, then .....

(a) $\frac{dy}{dx} = 2x + 2y$ (b) $2x + 2y = 1$ (c) $\frac{dy}{dx} = \frac{2x}{1-2y}$ (d) $\frac{dy}{dx} = \frac{x^2}{y - x^2}$

(21) If $f(x) = x^2 + 3$, then $\frac{d}{dx}\left(f(f(x))\right)$ ..... at $x = 1$

(a) 1 (b) 4 (c) 8 (d) 16

(22) If $f(3 - 2x) = 3x^2 + 1$, then $\grave{f}(7)$ = ...

(a) – 12 (b) – 2 (c) 6 (d) 42

(23) If $f(x - 1) = x - 2\,g(x^2 + 3)$ and $\grave{f}(1) = 9$, then $\grave{g}(7)$ = ...

(a) – 2 (b) – 1 (c) 1 (d) 2

(24) If $g(x) = \sqrt{f(x)}$ where each of $f(x)$, $g(x)$ is a differentiable function and $f(3) = 4$, then $\grave{g}(3) : \grave{f}(3)$ = ...

(a) 1 : 4 (b) 1 : 16 (c) 4 : 1 (d) 16 : 1

(25) If $f(x) = x^2 + 3$, then $\grave{f}\left(f(x)\right)$ = ... at $x = 1$

(a) 1 (b) 4 (c) 8 (d) 16

(26) If $2f(3) = 5\grave{f}(3) = 10$, $\grave{h}(5) = 2$, then $(h \circ f)\grave{}(3)$ = ...

(a) 1 (b) 2 (c) 3 (d) 4

(27) If $f(x) = \sqrt{g(x)}$ and $\grave{f}(x) = \frac{1}{\sqrt{g(x)}}$, then $\grave{g}(x)$ = ...

(a) 1 (b) 2 (c) x (d) $\grave{f}(x)$

(28) The opposite figure represents a curve of the function f

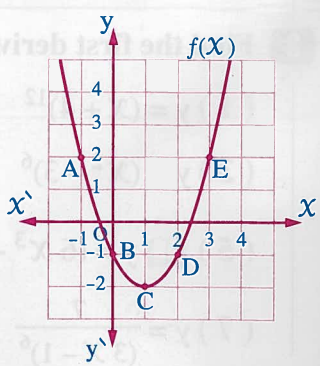
which of the following is positive .....

(a) $\frac{d}{dx}[f(x)]$ at A

(b) $\frac{d}{dx}\left[\left(f(x)\right)^2\right]$ at D

(c) $\frac{d}{dx}\left[\left(f(x)\right)^2\right]$ at B

(d) $\frac{d}{dx}\left[\left(f(x)\right)^3\right]$ at C



(29) In the opposite two figures :

The first figure represents a linear function and the second function represents a quadratic function

, then $\frac{dy}{dx}$ = ... at $z = 5$

(a) 4 (b) 8

(c) 12 (d) 16

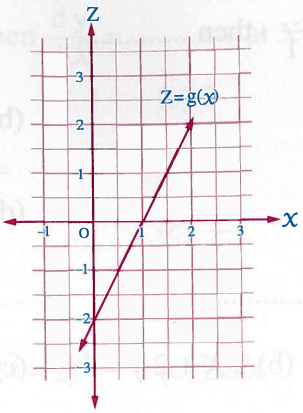


Fig (1)

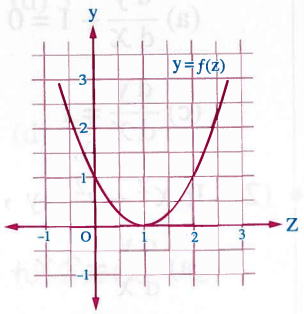


Fig (2)

Second Essay questions

1 Find $\frac{dy}{dx}$ of each of the following :

(1) $y = z^2$ and $z = 3x^2 - 2$ at $x = 2$ « 240 »

(2) $y = z^7$ and $z = x^3 + 2x^2 - 4$ at $x = 1$ « 49 »

(3) $y = z^5 + 3$ and $z = (x - 1)^3$ at $x = 2$ « 15 »

(4) $y = z^3 + z - 2$ and $z = x^2 - x$ at the point $(0 , -2)$ « -1 »

(5) $y = \sqrt{z}$ and $z = \frac{x-2}{x+1}$ at $x = 3$ « $\frac{3}{16}$ »

(6) $y = \frac{z-1}{z+1}$ and $z = \frac{x+1}{x-1}$ at $x = 2$ « $\frac{-1}{4}$ »

(7) $y = z^2 - z + 1$, $z = \sqrt[3]{(x-1)^2}$ at $x = 2$ « $\frac{2}{3}$ »

2 Find $\frac{dy}{dx}$ of each of the following :

(1) $y = 3z^2 - 1$, $z = \frac{5}{x}$

(2) $y = \sqrt[3]{z}$, $z = x^2 - 3x + 2$

(3) $y = z + \frac{1}{z}$, $xz = 1$

(4) $y = \frac{z^2}{1+z^2}$, $z = \sqrt{2x+1}$

3 Find the first derivative of each of the following :

(1) $y = (x + 4)^{12}$

(2) $y = (1 - 3x)^5$

(3) $y = (x^2 - 3)^6$

(4) $y = (3x^2 - 4x + 1)^4$

(5) $y = (6x^3 + 3x + 1)^{10}$

(6) $y = \left(x^2 + \frac{1}{x^2}\right)^7$

(7) $y = \frac{7}{(3x-1)^6}$

(8) $y = \frac{7}{(x^2-9)^5}$

4 **Find the first derivative of each of the following :**

(1) $y = x^5 (x+3)^7$ at $x = -2$ « – 144 »

(2) $y = (x-3)(x-5)^8$ at (4 , 1) « – 7 »

(3) $y = (1-x)^6 (1+x)^6$ at $x = \sqrt{2}$ « $12\sqrt{2}$ »

(4) 🕮 $y = (x^2+1)^5 (x^2-x+1)^{-4}$ at $x = 1$ « 32 »

(5) $y = \left(\frac{3x-2}{5x-4}\right)^9$ at $x = 1$ « – 18 »

(6) 🕮 $y = \left(\frac{x^2+1}{x-3}\right)^5$ at $x = 1$ « $\frac{-15}{2}$ »

(7) $y = \frac{x^7}{(3x-2)^4}$ at (1 , 1) « – 5 »

(8) $y = \frac{(x-1)^4}{(x^2+1)^5}$ at (0 , 1) « – 4 »

---

5 **If $y = f(x)$ find the first derivative of each of the following :**

(1) $y = \sqrt{x^2+1}$

(2) $y = \sqrt[3]{(2x+1)^5}$

(3) $y = \sqrt{9-2x^2}$ at $x = \frac{1}{\sqrt{2}}$ « $-\frac{1}{2}$ »

(4) 🕮 $y = \sqrt[3]{(2x^3+4x+3)^2}$ at $x = 2$ « $\frac{56}{9}$ »

(5) $y = x\sqrt{x-1}$ at (5 , 10) « $3\frac{1}{4}$ »

(6) $y = (x^2-1)\sqrt{2x+3}$ at $x = -1$ « –2 »

(7) $y = \frac{9x}{\sqrt{5x^2-4}}$ at (1 , 9) « –36 »

(8) $y = \frac{8x}{\sqrt[3]{x-2}}$ at $x = 3$ « 0 »

---

6 **Find each of the following :**

(1) $\frac{d}{dx}(y^5)$ (2) $\frac{d}{dy}(y^5)$

(3) $\frac{d}{dn}(y^5)$ (4) 🕮 $\frac{d}{dx}(y^n)$

7 **Find each of the following :**

(1) $\frac{d}{dx}\left(x\sqrt{2x^2+1}\right)$ at $x=0$ « 1 »

(2) $\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^4$ at $x=1$ « zero »

(3) $\frac{d}{dx}\left(x^2+2y^2\right)$

8 **Find $\frac{dy}{dx}$ of each of the following :**

(1) $y^5 = x^3 - 2x^2$

(2) 📖 $y^3 = 3x^2 - 4$

(3) $4y^2 = x^5 - 2x^3 + x$

(4) 📖 $y^3 = (x^2 - 4x + 4)^{-2}$

(5) $2\sqrt{y} = x^2 - 3$

(6) $x^2 - y^3 = 2x$

9 If $y = x^2 - 1$, $x = z^3 - 2$, find $\frac{dy}{dz}$, then prove that : $\frac{dy}{dz} + 4\frac{dx}{dz} = 6z^5$ « $6z^5 - 12z^2$ »

10 📖 If $y = z^2 + z$ and $z = 2x^2$, then prove that : $\frac{dy}{dx} - \frac{dz}{dx} = 16x^3$

11 If $y = \sqrt{1+x^2}$ and $z = 3x^2 - 7$, so prove that : $y\left(\frac{dy}{dx}\right) + \frac{dz}{dx} = 7x$

12 If $y = (x-5)^3$, then prove that : $\left(\frac{dy}{dx}\right)^3 - y(x-5)\frac{dy}{dx} = 24y^2$

13 If $x^3 - y^3 = 8$, then prove that : $\frac{dy}{dx} = \left(\frac{x}{y}\right)^2$

14 If $(x+y)^3 = 5$, then prove that : $\frac{dy}{dx} = -1$

15 📖 Find the values of x which make $\dot{f}(x) = 7$ where $f(x) = (x-5)^7$ « 6 or 4 »

16 Let $y = (ax+b)^2$ and let (2 , 1) lie on this curve and $\frac{dy}{dx} = 4$ at this point , so find a and b « ± 2 , ∓ 3 »

17 📖 **Volumes :** If the oil is poured at a rate of 10 cm^3/ sec in a cylindric barrel whose radius length of its base is 90 cm. , find the rate of rising the oil in the barrel with respect to the time. « $\frac{1}{810\pi}$ cm./sec. »

Third Higher skills

1 **Choose the correct answer from those given :**

(1) If $f(2) = \grave{f}(2) = 3$, and $g(x) = \left(x^2 f(x)\right)^2$, then $\grave{g}(2) =$ .....

(a) 576 (b) 768 (c) 672 (d) 480

(2) If f is an even function and differentiable for all values of $x \in \mathbb{R}$ where f is not a constant function , then $\grave{f}(x)$ is ..... function.

(a) an even (b) an odd

(c) neither odd nor even (d) both odd and even

(3) If $f(x) = 2x - 5$, then $\frac{d}{dx}\left(f^{-1}(x)\right) =$ .....

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

(4) If $f(x) = x^2 + 2$, $h(x) = x + 2$, then $\frac{d}{dx}\left((f \circ h)(x)\right) =$ ..... at $x = 1$

(a) 5 (b) 6 (c) 8 (d) 10

(5) If $k(x) = f\left(g(x)\right)$ where $f(-2) = 8$, $\grave{f}(-2) = 4$, $\grave{f}(5) = 3$, $g(5) = -2$, $\grave{g}(5) = 6$, then $\grave{k}(5) =$ .....

(a) 24 (b) 20 (c) 12 (d) 8

(6) If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots}}}$, then $\frac{dy}{dx} =$ .....

(a) 1 (b) $\frac{1}{xy}$ (c) $\frac{1}{2y+1}$ (d) $\frac{1}{2y-1}$

(7) The derivative of x^6 with respect to x^3 is .....

(a) x^3 (b) $2x^3$ (c) $3x^3$ (d) $6x^6$

---

2 If $y = \frac{2x}{\sqrt{1+x^2}}$, $z = \frac{\sqrt{1+x^2}}{5x}$, prove that : $5z^2 \frac{dy}{dx} + 2\frac{dz}{dx} = 0$

---

3 If $y = \left(\frac{x^2-2}{x^2+2}\right)^n$, prove that : $\frac{dy}{dx} = \frac{8nxy}{x^4-4}$

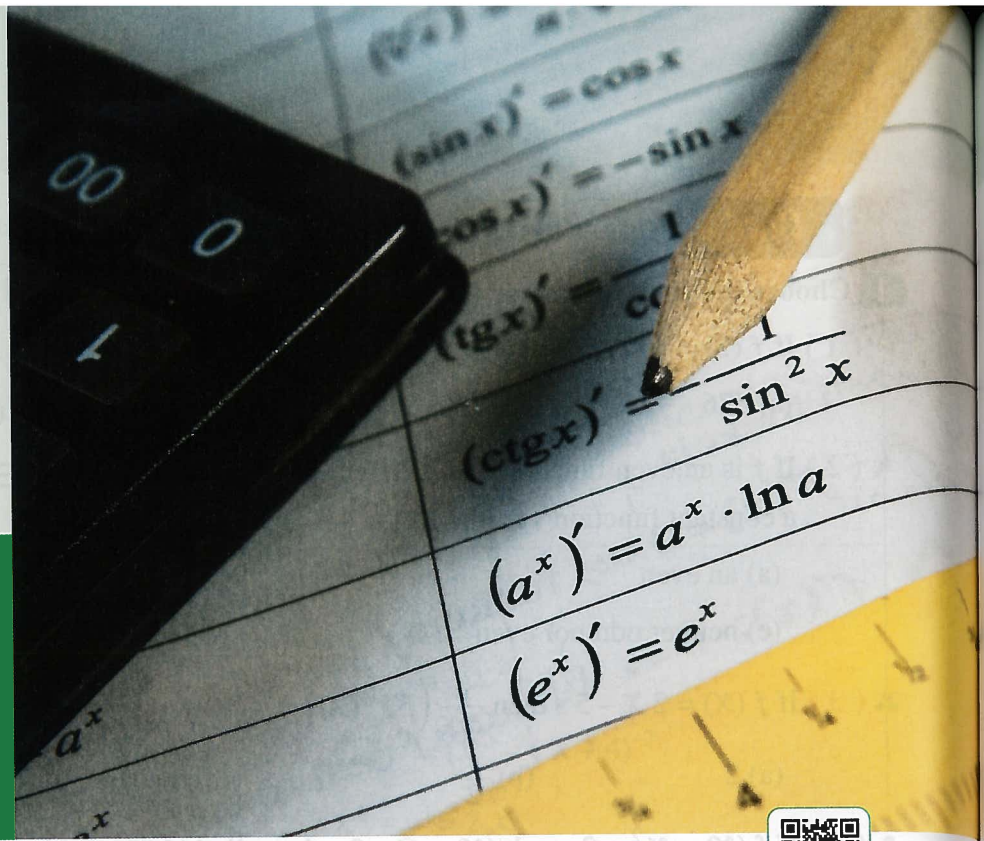
---

4 If $\frac{2}{\sqrt{x}+\sqrt{y}} = 5$, prove that : $\left(\frac{dy}{dx}\right)^2 = \frac{y}{x}$

Exercise

13

Derivatives of trigonometric functions



From the school book | Understand | Apply | Higher Order Thinking Skills

Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $f(x) = \tan(5x - \pi)$, then $\grave{f}\left(\frac{\pi}{4}\right) =$ .....

(a) 5 (b) $5\sqrt{2}$ (c) 10 (d) $10\sqrt{3}$

(2) If $y = \sin 2x$, then $\frac{dy}{dx} =$ ..... at $x = \frac{\pi}{6}$

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{3}$

(3) $\frac{d}{dx}(\cos^2 x + \sin^2 x) =$ .....

(a) 1 (b) zero (c) x (d) $2(\cos x + \sin x)$

(4) $\frac{d}{dx}\left(\tan\frac{\pi}{4}\right) =$ .....

(a) 1 (b) $\sec^2\frac{\pi}{4}$ (c) zero (d) 2

(5) If $y = \tan(9 - 7x)$, then $\frac{dy}{dx} =$ .....

(a) $\sec^2(9 - 7x)$ (b) $\sec^2(-7)$

(c) $-7\sec^2(9 - 7x)$ (d) $2\sec^2(9 - 7x)$

(6) If $y = \sin 4x$, then $\frac{dy}{dx} =$ .....

(a) $4\sin 3x$ (b) $4\sin 4x$ (c) $4\cos 4x$ (d) $-4\cos 4x$

(7) If $y = \cos\frac{1}{2}x$, then $\frac{dy}{dx} =$ .....

(a) $-\sin\frac{1}{2}x$ (b) $-\frac{1}{2}\sin\frac{1}{2}x$ (c) $\frac{1}{2}\cos x$ (d) $\frac{1}{2}\cos\frac{1}{2}x$

(8) If $y = 2 \sin (3x + 4)$, then $\dot{y}$ = ...

(a) $6 \cos (3x + 4)$ (b) $2 \cos (3x + 4)$

(c) $6 \cos (3x)$ (d) $-6 \cos (3x + 4)$

(9) If $y = \sin (x^2 + 3)$, then $\frac{dy}{dx}$ = ...

(a) $2 \cos (x^2 + 3)$ (b) $2 \cos x$

(c) $-2 \cos x$ (d) $2x \cos (x^2 + 3)$

(10) If $y = x^4 + \cos \frac{\pi}{2} - \sin \frac{x}{3}$, then $\frac{dy}{dx}$ = ...

(a) $4x^3 - \sin \frac{\pi}{2} - \frac{1}{3} \cos \frac{x}{3}$ (b) $4x^3 - \frac{1}{3} \cos \frac{x}{3}$

(c) $4x^3 - \sin \frac{\pi}{2} + \frac{1}{3} \cos \frac{x}{3}$ (d) $4x^3 - \frac{1}{3} \sin \frac{x}{3}$

(11) If $y = \cos \left(\frac{2}{x}\right)$, then $\dot{y}$ = ...

(a) $-\sin \left(\frac{2}{x}\right)$ (b) $\frac{-2}{x^2} \sin \left(\frac{2}{x}\right)$

(c) $\frac{2}{x^2} \sin \left(\frac{2}{x}\right)$ (d) $\frac{2}{x} \sin \left(\frac{2}{x}\right)$

(12) If $f(x) = \sin^2 x$, then $\dot{f}(x)$ = ...

(a) $\cos^2 x$ (b) $2 \cos^2 x$ (c) $2 \sin x$ (d) $2 \sin x \cos x$

(13) $\frac{d}{dx} (\tan 3x^2)$ = ...

(a) $6x \tan 3x^2 \sin^2 3x^2$ (b) $6 \tan 3x \sec^2 3x$

(c) $6x \sec^2 3x^2$ (d) $\sec^2 6x$

(14) If $y = x^3 + \cos 3x + \frac{1}{4} \sin 4x$, then $\frac{dy}{dx}$ = ... at x = zero

(a) zero (b) 1 (c) 3 (d) 4

(15) If $y = \cos \left(\frac{\pi}{2} - x\right) + \sin (\pi - 2x)$, then $\frac{dy}{dx}$ = ...

(a) $\sin x + \sin 2x$ (b) $\sin x + 2 \sin 2x$

(c) $\cos x + \cos 2x$ (d) $\cos x + 2 \cos 2x$

(16) If $y = \sin x$, then $\dot{y}$ = ...

(a) $y \cos x$ (b) $y \tan x$ (c) $y \cot x$ (d) $y \sin x$

(17) If $y = x \sin 2x$, then $\frac{dy}{dx}$ = ...

(a) $x \cos 2x + \sin 2x$ (b) $2x \cos 2x + \sin 2x$

(c) $-2x \cos 2x + \sin 2x$ (d) $-x \cos 2x + \sin 2x$

(18) $\frac{d}{dx}\left(\frac{1}{2}\cot 2x \sin 2x\right) =$ .....

(a) $-\sin 2x$ (b) $-\cos 2x$ (c) $\sin 2x$ (d) $\cos 2x$

(19) $\frac{d}{dx}\left(2\cos^2 x - 1\right) =$ .....

(a) $\sin 2x$ (b) $\cos 2x$ (c) $-2\sin 2x$ (d) $-2\cos 2x$

(20) If $y = 2x \tan 3x$, then $\frac{dy}{dx} = \frac{\text{.....}}{\cos^2 3x}$

(a) $x - \sin 6x$ (b) $2x + \sin 3x$

(c) $2x + \sin 6x$ (d) $6x + \sin 6x$

(21) If $y = 2\sin\frac{1}{2}x - x\cos\frac{1}{2}x$, then $\frac{dy}{dx} =$ ..... at $x = \pi$

(a) zero (b) 1 (c) $\frac{\pi}{2}$ (d) π

(22) If $y = \frac{x}{\cos 3x}$, then $\frac{dy}{dx} =$ ..... at $x = \frac{\pi}{3}$

(a) $\frac{1}{3\pi}$ (b) 1 (c) $\frac{\pi}{3}$ (d) -1

(23) If $y = \cos x$, then $y - \grave{y} =$ .....

(a) $\sin x - \cos x$ (b) $\sin x - \sin x$

(c) $\sin x + \cos x$ (d) zero.

(24) If $y = \sin x$, then $\frac{dy}{dx} =$ .....

(a) $\sin\left(\frac{\pi}{2} + x\right)$ (b) $\sin\left(x - \frac{\pi}{2}\right)$

(c) $\sin\left(\frac{3\pi}{2} + x\right)$ (d) $\sin\left(\frac{3\pi}{2} - x\right)$

(25) If $y = \sin^2(270° - x)$, then $\frac{dy}{dx} =$ .....

(a) $\sin 2x$ (b) $-\sin 2x$ (c) $\cos 2x$ (d) $-\cos 2x$

(26) If $y = \frac{\sin x}{1 + \cos x}$, then $\frac{dy}{dx} =$ .....

(a) $y \sin x$ (b) $y \cos x$ (c) $y \csc x$ (d) $y \sec x$

(27) If $y = \tan x$, then $\frac{dy}{dx} =$ .....

(a) $1 + y$ (b) $1 - y$ (c) $1 + y^2$ (d) $1 - y^2$

(28) If $y = (\sin x + \cos x)^2$, then $\frac{dy}{dx} =$ .....

(a) $2\sin 2x$ (b) $2\cos 2x$ (c) $2\sin x$ (d) $\cos x - \sin x$

(29) If $y = \sin^2 x - \cos^2 x$, then $\frac{dy}{dx}$ = ...

(a) $2 \sin 2x$ (b) $2 \cos 2x$ (c) $2 \sin x - 2 \cos x$ (d) $\cos x - \sin x$

(30) If $y = \tan^2 3x$, then $\dot{y}$ = ...

(a) $2 \tan 3x$ (b) $6 \tan 3x$ (c) $6 \tan 3x \sec^2 3x$ (d) $3 \sec^4 3x$

(31) If $y = \sec x$, then $\dot{y}$ = ...

(a) $\csc x$ (b) $\sec x \tan x$ (c) $\frac{1}{\sin x}$ (d) $\frac{-1}{\cos 2x}$

(32) If $y = \cos\left(\sqrt{x}\right)$, then $\frac{dy}{dx}$ = ... at $x = \frac{1}{4}\pi^2$

(a) $\frac{1}{\pi}$ (b) $\frac{-1}{\pi}$ (c) $\frac{-2}{\pi}$ (d) $\frac{2}{\pi}$

(33) If $y = \sin^3 4x$, then $\frac{dy}{dx}$ = ... at $x = \frac{\pi}{16}$

(a) 3 (b) 6 (c) $3\sqrt{2}$ (d) $6\sqrt{2}$

(34) If $xy + \sin x = 5$, then $\frac{dy}{dx}$ = ... at $x = \frac{\pi}{2}$

(a) $\frac{-4}{\pi}$ (b) $\frac{-8}{\pi}$ (c) $\frac{-4}{\pi^2}$ (d) $\frac{-16}{\pi^2}$

(35) If $y = x^2 \sin x$, then $\frac{dy}{dx}$ = ... at $x = \frac{\pi}{2}$

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) $\frac{-\pi}{2}$

(36) $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x\right)$ = ...

(a) $\sin^3 x$ (b) $-\sin^3 x$ (c) $\cos^3 x$ (d) $-\cos^3 x$

(37) If $y = 2 \sin\frac{x}{2} \cos\frac{\pi}{2}$, then $\dot{y}$ = ...

(a) $-\sin x$ (b) $\cos x$ (c) $\sin x$ (d) zero

(38) If $y = 16 \sin\frac{x}{4} \cos\frac{x}{4} \cos\frac{x}{2} \cos x$, then $\frac{dy}{dx}$ = ...

(a) $\frac{1}{2}\sin 2x$ (b) $\cos 2x$ (c) $2 \sin 2x$ (d) $4 \cos 2x$

(39) If $f(x) = \frac{\cos 2x}{\sin x - \cos x}$, then $\dot{f}\left(\frac{3\pi}{4}\right)$ = ...

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) zero

(40) If $y = \sqrt{\cos(5x)}$, then $\frac{dy}{dx}$ = ...

(a) $\frac{-5 \sin(5x)}{2\sqrt{\cos(5x)}}$ (b) $\frac{-\sin(5x)}{2\sqrt{\cos(5x)}}$

(c) $\frac{5}{2}\left(\cos(5x)\right)^{-\frac{1}{2}}$ (d) $\frac{1}{2}(\cos 5x)^{-\frac{1}{2}}$

(41) If $y = \frac{\sin^2 x}{1 + \cos x}$, then $\frac{dy}{dx}$ = ...

(a) $1 - \cos x$ (b) $\sin^2 x$ (c) $\sin x$ (d) $1 + \cos x$

(42) If $y = \sin(\cos^2 x)$, then $\dot{y}$ = ...

(a) $\cos(\cos^2 x)$ (b) $\cos(\cos^2 x) \times 2 \cos x$

(c) $\cos^3 x \times -2 \cos x$ (d) $-2 \sin x \cos x \cos(\cos^2 x)$

(43) If $y = \frac{x}{\sin x}$, then $\left[\frac{dy}{dx}\right]_{\left(x = \frac{3\pi}{2}\right)}$ = ...

(a) $\frac{3}{2}$ (b) $\cos\frac{3\pi}{2}$ (c) $\cos\pi$ (d) 1

(44) If $f(x) = \sqrt{\sin x + \cos x}$, then $\dot{f}(0)$ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) -1

(45) If $f(x) = \sin^4 x$, then $\dot{f}\left(\frac{\pi}{4}\right)$ = ...

(a) $\frac{1}{2}$ (b) 4 (c) $\frac{1}{4}$ (d) 1

(46) The derivative of $1 - \cos^2 x$ with respect to $\cos^2 x$ is .....

(a) -1 (b) 1 (c) $\tan x$ (d) $-\tan x$

(47) If $f(x) = a \sin x$, $g(x) = \frac{\tan x}{b}$ where a and b are constants and $\dot{f}(\pi) = \dot{g}\left(\frac{\pi}{3}\right)$, then a b = ...

(a) -4 (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) 4

Second Essay questions

1 Find the first derivative of each of the following :

(1) $y = 5 \sin(3 - \pi x)$

(2) $y = \cos(5x + 3)$

(3) $y = 3 \tan(2x + 3)$

(4) $y = \tan(-5x^2 + 19)$

(5) $y = 2 \cos(4 - 3x^2)$

(6) $y = x - \sin 2x + \tan 5x$

(7) $y = \sin\frac{\pi}{4} - 7 \sin x$

(8) $y = \cos(2x - 1) + \sin(4 - 5x)$

(9) $y = \tan\left(\sqrt{x}\right)$

(10) $y = \sin\left(\frac{1}{x^2}\right)$

2 Find the first derivative of each of the following :

(1) $y = x \sin(3x - 2)$

(2) $y = x^2 \sin 2x$

(3) $y = x \tan(3x) + \cos x$

(4) $y = x \sin x + \sqrt{x}$

(5) $y = 2 \sin 2x \cos 3x$

(6) $y = 2 \sin (2x + 1) \cos (2x + 1)$

(7) 📖 $y = x^2 \sin (2x^2 + 5)$

(8) 📖 $y = \frac{\tan x}{x}$

(9) $y = \frac{\sin 2x}{\cos x}$

(10) $y = \frac{1 + \cos 2x}{1 - \cos 2x}$

(11) $y = \cot x$

(12) $y = \csc x$

3 **Find the first derivative of each of the following :**

(1) $y = \tan^4 \frac{1}{2} x$

(2) 📖 $y = \cos^2 (4x^2 - 7)$

(3) 📖 $y = 4 \cos^5 x$

(4) 📖 $y = \cos^3 \left(\frac{x}{x+1}\right)$

(5) $y = \tan^3 (4x - 7 - 2x^3)$

(6) $y = x\sqrt{\sin x}$

(7) $y = 2 \cos x \cos \frac{x}{2} \sin \frac{x}{2}$

(8) $y = \frac{\sin 2x}{1 - \cos 2x}$

(9) $x = y \cos x$

(10) $y = x^5 \cos \frac{\pi}{x}$

(11) $x^2 = y \sec (3x - 2)$

(12) 📖 $y = \cos (\cos x)$

(13) 📖 $y = \sin (\tan 3x)$

(14) 📖 $y = \sec^2 x - 1$

(15) $y = \sqrt{\tan^5 x}$

(16) $y = \frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}}$

4 **Find $\frac{dy}{dx}$ in each of the following :**

(1) $y = \sin 2x \cos x$ at $(\pi , 0)$ « -2 »

(2) $y = (x - 5) \left(\cos \frac{\pi x}{8}\right)$ at $x = 4$ « $\frac{\pi}{8}$ »

(3) $y = \sin x \tan 4x$ at $x = \frac{\pi}{4}$ « $2\sqrt{2}$ »

(4) $y = \sqrt{1 + \cos x}$ at $x = \frac{\pi}{2}$ « $-\frac{1}{2}$ »

(5) $y = \frac{x^3}{\cos x}$ at $x = \pi$ « $-3\pi^2$ »

(6) $y = \frac{2 \cos 3x}{x + 1}$ at $x = 0$ « -2 »

(7) $y = (1 + \sin x)(1 - \cos x)$ at $x = \pi$ « -2 »

(8) $y = (\sin 2x - \cos 4x)^4$ at $x = 0$ « -8 »

5 📖 If $y = 2x \sin x \cos x$, then prove that : $\frac{dy}{dx} = \sin 2x + 2x \cos 2x$

6 If $y = x \sin x$, then prove that : $x \left(\frac{dy}{dx} - x \cos x\right) = y$

7 If $y = \frac{1}{3}\cos^3 x - \cos x$, then prove that : $\frac{dy}{dx} = \sin^3 x$

8 If $y = \frac{\sin x}{1+\cos x}$, then prove that : $(1+\cos x)\frac{dy}{dx} = 1$

9 If $y = \frac{\sin x}{\sin x + \cos x}$, then prove that : $\frac{dy}{dx} = \frac{1}{1+\sin 2x}$

10 If $y = \frac{1+\tan x}{1-\tan x}$, then prove that : $\frac{dy}{dx} = \sec^2(x+45°)$

11 If $y = \frac{\sin x}{1+\sin x}$, then prove that : $\frac{1}{y^2} \times \frac{dy}{dx} = \cot x \csc x$

12 If $y = \frac{1+\sin^2 x}{1-\sin^2 x}$, then prove that : $\frac{dy}{dx} = 4\tan x \sec^2 x$

13 If $2x + y = \sin 2x$, then prove that : $\frac{dy}{dx} + 4\sin^2 x = 0$

14 If $y = \sin x + \cos x$, then prove that : $(\dot{y})^2 + y^2 = 2$

15 Find $\frac{dy}{dx}$ If $y = \sin x$ where x is measured by the degree measurement.

16 If $y = \sin^2 x - \cos 2x - 4x$:

(1) Find the rate of change of y with respect to the variable x.

(2) Find the values of $x \in]0 , \pi[$ when the rate of change equals – 1 « $\frac{\pi}{4}$ »

17 If $y = \sec 4x$, then find rate of change of y with respect to x when $x = \frac{\pi}{4}$ « zero »

18 Prove that :

(1) $\frac{d}{dx}(\sin^2 x) = -\frac{d}{dx}(\cos^2 x)$

(2) $\frac{d}{dx}(\sin^2 x + \cos^2 x) = \frac{d}{dx}(\sec^2 x - \tan^2 x)$

(3) $\frac{d}{dx}\left(\frac{1}{2}\tan x \sin 2x\right) = \sin 2x$

19 Find $\frac{dy}{dx}$ for each of the following :

(1) $y = z - \sin z$ and $z = 5x + \pi$ at $x = 0$ « 10 »

(2) $y = (3z+1)^5$ and $z = \frac{1}{3}\cos 2x$ at $x = \frac{\pi}{4}$ « –10 »

(3) $y = \sqrt{4-2z}$ and $z = \tan 2x$ at $x = \frac{\pi}{8}$ « $-2\sqrt{2}$ »

Third Higher skills

1 Choose the correct answer from those given :

(1) If $\sin x \cos y - \sin y \cos x = \frac{1}{2}$, then $\frac{dy}{dx} =$ .....

(a) zero (b) $\sin(x + y)$ (c) $-\cos(x + y)$ (d) 1

(2) If $y = \frac{2\tan x}{1 - \tan^2 x}$, then $\frac{dy}{dx} =$ .....

(a) $2\tan 2x$ (b) $2\sec^2 2x$ (c) 1 (d) zero

(3) $\frac{d}{dx}\left[x^2 + \frac{d}{dx}(x + \sin x)\right] =$ .....

(a) $\cos 2x - \sin x$ (b) $2x - \sin x$

(c) $2x + 1 + \cos x$ (d) $2x + \sin x$

(4) $\lim_{x \to a} \frac{\sin x - \sin a}{x - a} =$ .....

(a) $\cos x$ (b) 1 (c) $\cos a$ (d) $\sin(x - a)$

(5) $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{4} + h\right) - \sin\left(\frac{\pi}{4}\right)}{h} =$ .....

(a) $\frac{\sin h}{h}$ (b) $\sin h$ (c) $\cos h$ (d) $\cos\frac{\pi}{4}$

(6) If $y = \cos\sqrt{x}$, then $\lim_{h \to 0}\left(\frac{dy}{dx}\right) =$ .....

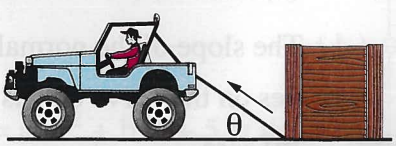
(a) $-\sin\sqrt{x}$ (b) $\cos\sqrt{x}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

---

2 If $y = z^4 - 2z^2 + 1$ and $z = \sin 4x$, then prove that : $\frac{dy}{dx} + 16y\tan 4x = 0$

---

3 Mechanics : A force of a magnitude F has acted on an object of weight (w) in the direction that creates an angle of measurement θ with the direction of the motion and the magnitude of the force is given by the rule $F = \frac{\mu w}{\mu \sin\theta + \cos\theta}$



Where μ is constant called the friction coefficient

(1) Find the rate of change of the force F with respect to the angle θ

(2) Write the necessary condition so that the rate of change equal zero ?

Exercise 14

Applications on derivatives

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The slope of the tangent to the curve of the function $y = (2x - 3)^5$ at $x = 2$ equals .....

(a) 1 (b) $\frac{1}{12}$ (c) 5 (d) 10

(2) The slope of the tangent to the curve of the function $y = \sin x \cos x$ equals .....

(a) $\cos x \sin x$ (b) $\cos^2 x - \sin^2 x$

(c) $\sin x - \cos x$ (d) $\sin^2 x + \cos^2 x$

(3) The slope of the tangent to the curve $y = x \cos 2x$ at $x = \frac{\pi}{2}$ equals .....

(a) zero (b) 1 (c) -1 (d) 2

(4) The slope of the normal to the curve $y = \sin 2x$ at the point with x-coordinate $\frac{\pi}{6}$ and lies on the curve equals .....

(a) zero (b) -1 (c) 3 (d) 1

(5) If the straight line $y + x - 1 = 0$ touches the curve of the function $f : f(x) = x^2 - 3x + a$, then a = ...

(a) 1 (b) 2 (c) 3 (d) 4

(6) The point on the curve of the function $f : f(x) = (x - 3)^2 - 1$ at which the tangent is parallel to the straight line : $2x + y - 3 = 0$ is .....

(a) (0 , 2) (b) (2 , 0) (c) (3 , -1) (d) (1 , 3)

(7) The point on the curve of the function $y = \frac{1}{x-3}$ at which the tangent to the curve parallel to the line $x + y = 0$ from the following is .....

(a) $\left(-2 , \frac{-1}{5}\right)$ (b) $(4 , -4)$ (c) $(4 , 1)$ (d) $(2 , -2)$

(8) If f is a linear function and $\grave{f}(-11) = 17$, then $\grave{f}(11) =$ .....

(a) –11 (b) –17 (c) 17 (d) 11

(9) If the tangent to the curve of the function $y = f(x)$ at the point $(3 , 5)$ passes through the point $(6 , -1)$, then $\grave{f}(3) =$ .....

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

(10) The equation of the tangent to the curve of the function $f : f(x) = \frac{1}{x+1}$ at $x =$ zero is .....

(a) $y = 1 - x$ (b) $y = x - 1$ (c) $y = x + 1$ (d) $y = -x - 1$

(11) The equation of the tangent to the curve $y = \tan x + \sin 2x$ at $x = \frac{\pi}{4}$ is .....

(a) $2y - 4x - \pi = 0$ (b) $2x - 4y + \pi = 4$

(c) $y - 2x = 2$ (d) $2y - 4x + \pi = 4$

(12) The curve of the function $f : f(x) = \frac{1}{3}x^3 - x^2 + 3$ has a horizontal tangent at $x =$ .....

(a) zero (b) zero or 3 (c) 2 (d) zero or 2

(13) The tangent to the curve $y = \sqrt[3]{x}$ at $x = 0$ is parallel to .....

(a) x-axis (b) y-axis

(c) the line $y = x$ (d) the line $x + y = 0$

(14) If the equation of the normal to the curve $f(x)$ at the point $(2 , -1)$ is $x - 2y = 4$, then $\grave{f}(2) =$ .....

(a) 2 (b) -2 (c) 1 (d) -1

(15) If the tangent to the curve of the function f where $f(x) = ax^2 + bx + 5$ at the point $(-1 , 3)$, which lies on it , makes with the positive direction of the x-axis a positive angle of measure 45° , then $a + b =$ .....

(a) 1 (b) 2 (c) 3 (d) 4

(16) If the slope of the tangent to the curve : $y = ax^2 + 2bx$ at the origin equals 6 and the point $(-1 , -3)$ lies on the curve , then $a + b =$ .....

(a) 3 (b) 4 (c) 6 (d) 8

(17) If $y = \frac{1}{a + b x}$ is the equation of a curve passes through the point $(1 , -1)$ and has a slope of 2 at this point , then $b - a =$ .....

(a) -1 (b) -3 (c) 3 (d) 1

(18) The positive measure of the angle that the tangent to the curve : $y + \sin 2x = 0$ makes with the positive direction of the x-axis at the point $\left(\frac{\pi}{3} , \frac{-\sqrt{3}}{2} \right)$ equals .....

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

(19) Slope of tangent to the curve $y = \sqrt[3]{x^2 + x + 2}$ at the point $(2 , 2)$ equals .....

(a) $\frac{2}{3}$ (b) $\frac{12}{5}$ (c) $\frac{3}{2}$ (d) $\frac{5}{12}$

(20) Slope of normal to the curve $y = (x - 1)(x^3 + 2)$ at $x = 1$ equals .....

(a) $-\frac{1}{3}$ (b) -3 (c) $\frac{1}{3}$ (d) 3

(21) Measure of the positive angle that the tangent to the curve $y = \sqrt{2x^2 + 7}$ at the point $(-3 , 5)$ makes with the positive direction of x-axis $\simeq$ .....

(a) $39° 48\grave{}$ (b) $50° 12\grave{}$ (c) $129° 48\grave{}$ (d) $140° 12\grave{}$

(22) Equation of the normal to the curve $y = \tan x$ at the point $\left(\frac{\pi}{4} , 1 \right)$ which lies on it is .....

(a) $y - 2x = 1 - \frac{\pi}{2}$ (b) $8y + 4x = \pi + 8$

(c) $4y - 4x = \pi + 4$ (d) $4y + 2x = \pi + 4$

(23) The point lies on the curve $y = x^2$ at which slope of tangent equals x-coordinate of that point , then the point is .....

(a) $(2 , 0)$ (b) $(0 , 2)$ (c) $\left(-\frac{1}{2} , \frac{1}{2} \right)$ (d) $(0 , 0)$

(24) The point that lies on the curve $y = x^3 - 2x + 1$ at which tangent of the curve makes an angle of measure $\frac{\pi}{4}$ with the positive direction of x-axis is .....

(a) $(1 , 0) , (1 , 2)$ (b) $(0 , 1) , (2 , -1)$

(c) $(1 , 0) , (-1 , 2)$ (d) $(1 , 2) , (-1 , 2)$

(25) The point lies on the curve $y = 2x^3 - x + 3$ at which the tangent is perpendicular to the straight line $x = 1 - 5y$ is .....

(a) $(1 , -4)$ (b) $(1 , 2)$

(c) $(1 , -4) , (1 , 2)$ (d) $(1 , 4) , (-1 , 2)$

(26) The points on the curve $y^2 = 8x$ at which $\frac{dy}{dx} = \frac{dx}{dy}$ are .....

(a) $(2, 4), (4, -2)$ (b) $(2, 4), (2, -4)$

(c) $(-2, 4), (2, 4)$ (d) $(4, 2), (4, -2)$

(27) If the straight line : $2y - 4x + a = 0$ touches the curve of the function : $y = x^2 + 1$ at the point (b, c), then $a + b + c =$ .....

(a) 1 (b) 2 (c) 3 (d) 6

(28) If the tangent to the curve : $y = x^3 - 3x^2$ makes an obtuse angle with the positive direction of x-axis, then $x \in$ .....

(a) $[0, 2]$ (b) $]0, 2[$ (c) $\mathbb{R} - [0, 2]$ (d) $\mathbb{R} -]0, 2[$

(29) If the slope of the tangent to the curve $y = x^2 + ax + b$ equals -1 at the point $(2, -2)$, then $a \times b =$ .....

(a) -15 (b) -20 (c) -1 (d) 10

(30) The tangent to the curve $y = \cos x$ at $x = \frac{\pi}{2}$ makes with the positive x-axis a positive angle of measure .....

(a) 2 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{4}$

(31) 📖 The tangent to the curve $y = (3x - 5)^3$ at the point $(2, 1)$ makes with the positive direction of x-axis a positive angle whose tangent equals .....

(a) 2 (b) 3 (c) 7 (d) 9

(32) If $y = \frac{2\tan x}{1 - \tan^2 x}$, then the slope of the tangent at $x = \frac{\pi}{8}$ equals .....

(a) 2 (b) 4 (c) 6 (d) 8

(33) If the perpendicular to the curve of the function $y = f(x)$ at the point $(3, 4)$ makes with the positive x-axis an angle of meausre $\frac{3\pi}{4}$, then $\dot{f}(3) =$ .....

(a) -1 (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) 1

(34) 📖 If the straight line $y = 8 - 3x$ is a tangent to the curve of the function f at the point $(3, -1)$, then $\dot{f}(3) =$ .....

(a) -1 (b) -3 (c) 3 (d) 8

(35) The slope of the perpendicular to the curve of the function : $y = |x^3|$ at the point $(-2, 8)$ is .....

(a) 12 (b) -12 (c) $\frac{1}{12}$ (d) $\frac{-1}{12}$

(36) If f is an even differentiable function on $\mathbb{R}$ and $\dot{f}(2) = 3$, then slope of tangent of f at $x = -2$ is .....

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(37) If f is an odd differentiable function on $\mathbb{R}$ and $\grave{f}(3) = 5$, then slope of tangent at $x = -3$ is .....

(a) 5 (b) -5 (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

(38) Tangent to the curve is perpendicular to x-axis if .....

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) $\frac{dx}{dy} = -1$

(39) The two tangents to the curve $y = x^3$ at $x_1 = -1$ and $x_2 = 1$ are .....

(a) perpendicular. (b) parallel.

(c) intersecting and not perpendicular. (d) coincident.

(40) If the slope of tangent to the curve of the function f where $f(x) = a \sin^2 x + b \cos^2 x$ at the point $(\frac{\pi}{3}, 1)$ equals $6\sqrt{3}$, then $a + b =$ .....

(a) -12 (b) -4 (c) 8 (d) 12

(41) If the two curves $y = x^3$, $a y + x^2 = b$ are perpendicular intersecting at $(1, 1)$, if $a =$ .....

(a) -2 (b) zero. (c) 3 (d) 6

(42) If the curve of the function f passes through the point $(3, 7)$ at which slope of tangent is -2 and $g(x) = (x + 2) f(x) + 2x - 3$, then measure of the angle that the tangent to the curve of g at $x = 3$ makes with the positive direction of x-axis equals .....

(a) zero. (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

(43) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3 - a x^2 + b x - 1$ and tangent at $x = 1$ makes an angle of measure 135° with the positive direction of x-axis and the tangent at $x = 2$ is parallel to x-axis , then $3a - b =$ .....

(a) -8 (b) -4 (c) zero (d) 8

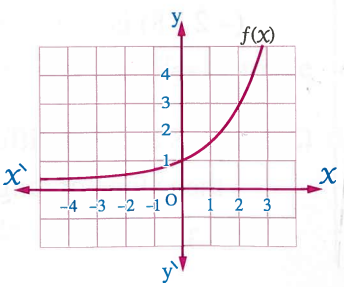
(44) Area of the included triangle between tangent to the curve $y = \frac{4}{x}$ where $x > 0$ at any point on it and the two coordinate axes equals ..... square unit.

(a) 2 (b) 4 (c) 8 (d) 16

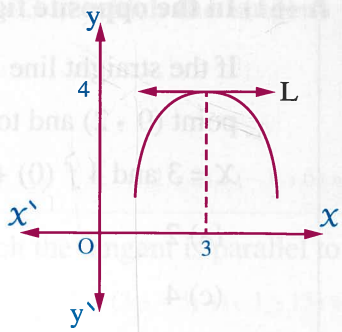
(45) The given figure represents the curve of the function f , then $\grave{f}(2)$ is .....

(a) positive. (b) negative.

(c) zero. (d) undefined.



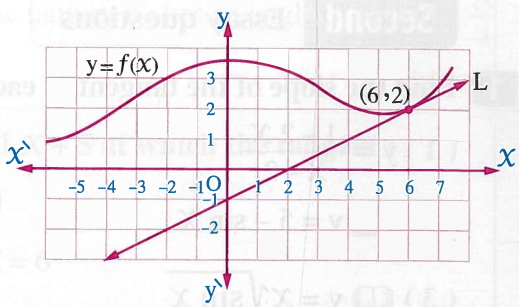
(46) The opposite graph represents a part of the curve of the function f and the straight line L is a tangent to the curve at the point $(3 , 4)$ and parallel to X-axis , then $\grave{f}(3) =$ .....



(a) 4 (b) zero.

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

(47) The opposite figure represents the curve of the function $y = f(X)$, and the straight line L touches the curve at the point $(6 , 2)$, then $\grave{f}(6) =$ .....

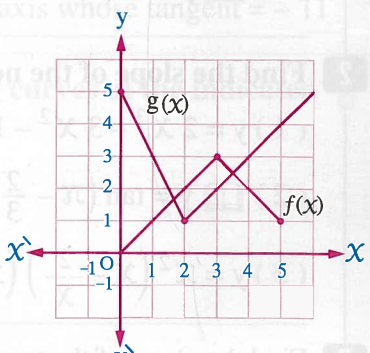


(a) $-\frac{1}{2}$ (b) zero

(c) $\frac{1}{2}$ (d) 1

(48) In the opposite figure :

$\grave{f}(1) + \grave{g}(1) =$ .....



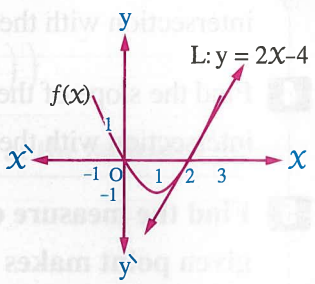
(a) – 2

(b) – 1

(c) 1

(d) 2

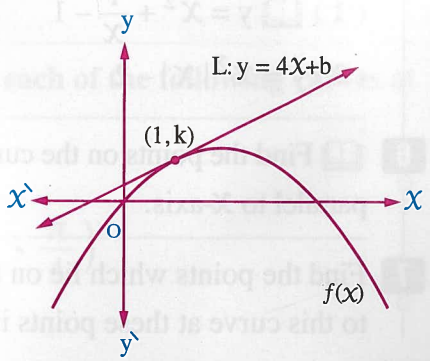
(49) The opposite figure shows the curve of the function f , the straight line L touches the curve at $(2 , 0)$, then $\grave{f}(2) =$ .....



(a) zero (b) 1

(c) 2 (d) 3

(50) The opposite figure shows the curve of the function $f : f(X) = aX - X^2$ and the straight line L touches the curve at point $(1 , k)$, then $a =$ .....

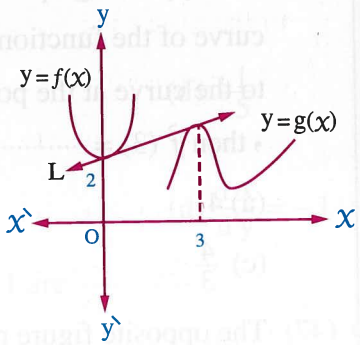


(a) – 3 (b) zero

(c) 6 (d) 8

(51) In the opposite figure :

If the straight line L touches the curve $y = f(x)$ at point $(0 , 2)$ and touches the curve $y = g(x)$ at $x = 3$ and $4 \grave{f}(0) + 5 \grave{g}(3) = 3$, then $g(3) =$ .....



(a) 2 (b) 3

(c) 4 (d) 5

Second Essay questions

1 Find the slope of the tangent of each of the following curves :

(1) $y = \frac{1-2x}{x-2}$ at the point $(1 , 1)$ « 3 »

(2) $y = 5 - \sin x$ at $x = \frac{\pi}{4}$ « $-\frac{1}{\sqrt{2}}$ »

(3) $y = x\sqrt{\sin x}$ at $x = \frac{\pi}{2}$ « 1 »

2 Find the slope of the normal to each of the following curves :

(1) $y = 2x^3 - 3x^2 - 12x + 5$ at the point $(-2 , 1)$ « $\frac{-1}{24}$ »

(2) $y = \tan\left(\pi - \frac{2}{3}x\right)$ at the point $\left(\pi , \sqrt{3}\right)$ « $\frac{3}{8}$ »

(3) $y = x^2\left(x - \frac{2}{x}\right)\left(x + \frac{2}{x}\right)$ at $x = 2$ « $\frac{-1}{32}$ »

3 Find the slope of the tangent to the curve : $y = (x + 2)(x^3 + 1)$ at the points of its intersection with the x-axis. « $-7 , 3$ »

4 Find the slope of the tangent to the curve : $f(x) = x^3 - 2x^2 + 3x - 1$ at the point of its intersection with the y-axis. « 3 »

5 Find the measure of the positive angle which the tangent to the given curve at the given point makes with the positive direction of the x-axis :

(1) $y = x^2 + \frac{1}{x} - 1$ at $x = 1$ « 45° »

(2) $y = x^2|x| - 2$ at $x = -2$ « 94° 45̂ 49̂ »

6 Find the points on the curve : $y = x^3 - 6x^2 - 15x + 20$ at which the tangent is parallel to x-axis. « $(5 , -80) , (-1 , 28)$ »

7 Find the points which lie on the curve : $y = (x - 3)(x^2 + 2)$ and the slope of the tangents to this curve at these points is equal to 11 « $(3 , 0) , (-1 , -12)$ »

8 📖 **Find the points which lie on the curve of : $y = x^2 - 2x + 3$ at which the tangent to the curve is :**

(1) Parallel to x-axis

(2) Perpendicular to the straight line $x - 4y + 1 = 0$ « (1 , 2) , (– 1 , 6) »

9 Find the points on the curve : $y = x^3 - 3x^2 - 5x + 12$ at which the tangent is parallel to the line that passes through the two points : (3 , 1) and (5 , 9) « (3 , – 3) , (– 1 , 13) »

10 Find the points on the curve : $y = \frac{x-2}{x+2}$ at which the tangent makes an angle of measure $\frac{\pi}{4}$ with the positive direction of x-axis. « (0 , – 1) , (– 4 , 3) »

11 📖 **Find the points on the curve : $y = 3x^3 - 11x + 5$ at which the tangent :**

(1) Is parallel to the straight line $2x + y - 5 = 0$

(2) Is perpendicular to the straight line $25y + x = 6$

(3) Makes a positive angle with the positive direction of x-axis whose tangent = – 11

12 **Find the equation of the tangent for each of the following curves at the indicated points :**

(1) $y = 3x^3 - 7x^2 + x - 2$ at the point (1 , – 5)

(2) $y = \sqrt{x} + \frac{4}{\sqrt{x}}$ at the point (4 , 4)

(3) 📖 $y = 2 \sin x + \cos x$ at the point (0 , 1)

(4) 📖 $y = 2x \sin x \cos x$ at $x = \pi$

(5) 📖 $y = 4x - \tan x$ at the point $\left(\frac{\pi}{4} , f\left(\frac{\pi}{4} \right) \right)$

13 **Find the equation of the normal on each of the following curves at the indicated points :**

(1) $y = x^5 - 4x^3 + 2$ at $x = -1$

(2) 📖 $y = \frac{x^2 - 1}{2 - x^2}$ at $x = 0$

14 **Find the equations of the tangent and the normal on each of the following curves at the indicated points :**

(1) $y = \sin x \tan 6x$ at $x = \frac{\pi}{6}$

(2) 📖 $y = x \sin 2x$ at the point $\left(\frac{\pi}{4} , \frac{\pi}{4} \right)$

(3) $y = \sqrt{\sin x + \cos x}$ at $x = \frac{\pi}{2}$

15 Find the two equations of the tangent and the normal to the curve of $y = \frac{x+3}{x+1}$ at the point lying on the curve and whose abscissa = 1

Does the point A $(-3, 4)$ lie on the tangent ? Explain.

16 Find the equation of the tangent to the curve $y = 2x^3 + 6x^2 + 5x$ which makes a positive angle of measure 135° with the positive direction of x-axis. « $y + x + 2 = 0$ »

17 Find the equation of the tangent to the curve : $y^2 = x - 1$, if its slope $= -\frac{1}{2}$ « $2y + x = 0$ »

18 Find the equation of the tangent to the curve of the function $y = (x-2)(x+1)$ at each of its two intersection points with x-axis. « $y - 3x + 6 = 0$, $y + 3x + 3 = 0$ »

19 If the curve $y = ax^3 + bx^2$ touches the straight line $y = 8x + 5$ at the point $(-1, -3)$, find the two values of a and b « $2, -1$ »

20 If the curve $y = (x^2 - 2x)(ax + b)$ touches the x-axis at point $(2, 0)$, and touches the straight line $y = 2x$ at the origin point , find the two values of a and b « $\frac{1}{2}, -1$ »

21 If $x \in [0, \pi[$, then find the points on the curve $y = \sin 2x$ at which the tangent is parallel to the straight line $y = 8 + x$ « $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, \frac{-\sqrt{3}}{2}\right)$ »

22 Prove that the tangent to the curve $y = x^3 + 3x - 2$ at any point on it inclines at an acute angle to the x-axis , then find the equation of the normal to the curve at the point $(1, 2)$ on the curve. « $x + 6y - 13 = 0$ »

23 Prove that the tangent drawn to the curve of $y = x^2 + x - 1$ at the point $(1, 1)$ is perpendicular to the tangent drawn to the curve of $y = 2 - \sqrt[3]{x}$ at the same point.

24 Find the slope of the tangent of the curve : $4y = 12x - 4 - x^2$ at $x = 2$ and prove that it is double the slope of the curve at $x = 4$ also , find the point on the curve at which the slope of the tangent equals -1 « $(8, 7)$ »

25 Prove that the tangent to the curve : $y = 3x^2 - 5x + 2$ at $(1, 0)$ makes with the positive direction of the x-axis a positive angle of measure $\frac{\pi}{4}$

Find also the equation of this tangent. « $y = x - 1$ »

26 Prove that the tangent to the curve : $y = \sin x + \cos x$ at the point $\left(\frac{\pi}{4}, \sqrt{2}\right)$ is parallel to the x-axis and find its equation. « $y = \sqrt{2}$ »

27 Find the equation of the two tangents to the curve $y = x^3 - 3x + 5$ which are perpendicular to the straight line $x + 9y = 1$ « $y - 9x + 11 = 0$, $y - 9x - 21 = 0$ »

28 Find the equation of the tangent to the curve : $y = x^2 - 2x + 3$ at the point $(x , 3)$ which lies on it «explain the existence of two solutions» « $y + 2x - 3 = 0 , y - 2x + 1 = 0$ »

29 Find the equation of the tangent for the curve : $y = 5x - 2x^2 - 4$ at the point $(1 , -1)$ lying on it and if this tangent intersects the y-axis at the point A and intersects the x-axis at the point B , find the area of Δ OAB such that O $(0 , 0)$ « $y = x - 2$, 2 unit area »

30 Find the equation of the tangent to the curve : $y = \sqrt{25 - x^2}$ at the point A $(3 , 4)$ which lies on it and if this tangent intersects the x-axis at the point B , find the area of Δ BAO where O is the origin point. « $4y + 3x = 25$, $\frac{50}{3}$ unit area »

31 Prove that the area of the triangle bounded by the tangent to the curve $y = \frac{1}{x}$ where $x > 0$ at any point on it and the two coordinate axes equals 2 square units.

Third Higher skills

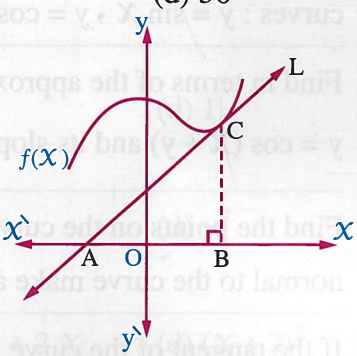
1 Choose the correct answer from those given :

(1) The rate of change of the slope of the tangent of the function $f : f(x) = 2x^3$ at $x = 3$ equals .....

(a) 30 (b) 32 (c) 34 (d) 36

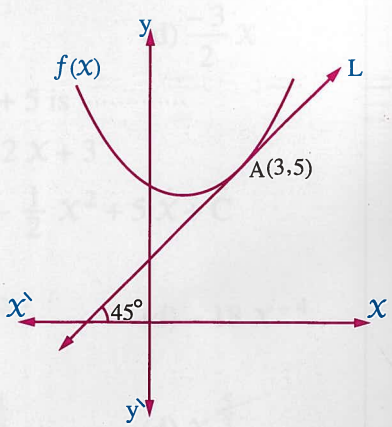
(2) In the opposite figure :

The straight line L is a tangent to the curve of the function f at the point C and cuts the x-axis at the point A $(-4 , 0)$ and B $(4 , 0)$, $f(4) + \grave{f}(4) = 9$, then the area of Δ ABC = ... square unit.



(a) 30 (b) 32 (c) 36 (d) 42

(3) The opposite figure represents the function f and the straight line L touches the curve of the function at the point A $(3 , 5)$ and $h(x) = x \cdot f(x)$, then $\grave{h}(3)$ = ...



(a) 3

(b) 1

(c) 5

(d) 8

(4) If the curves of the two functions f and g touch each other at $(2 , 3)$, $h(x) = x . f(x) - 2 g(x)$, then $\grave{h}(2) =$ .....

(a) -3 (b) -2 (c) 2 (d) 3

2 Find the surface area of the triangle formed from x-axis , the tangent and the normal to the curve of $y = x^2 - 6x + 13$ at point $(4 , 5)$ lying on it. « 31.25 unit area »

3 Find the equation of the tangent to the curve : $y + \sqrt{x} = 12$ at its intersection point with the line : $y = x$ « $x + 6y - 63 = 0$ »

4 If the tangent to the curve : $y = \frac{-x}{a + b x}$ at the point $(-1 , 1)$ makes with the positive direction of the x-axis a positive angle of measure θ where $\cos\theta = \frac{-1}{\sqrt{10}}$, find the values of a and b « 3 , 2 »

5 Prove that the two curves : $y = 3x^2 - 5x - 2$, $y = x^2 - 3x - 2$ intersect orthogonally at the point $(1 , -4)$

6 If $x \in [0 , \pi]$, find the measure of the acute angle between the two tangents of the two curves : $y = \sin x$, $y = \cos x$ at their intersection point. « $70° 31\grave{} 44\grave{}\grave{}$ »

7 Find in terms of the approximated value "π" The equation of the tangent to the curve : $y = \cos(x + y)$ and its slope $= \frac{-1}{2}$, where $x \in [0 , \pi]$ « $2x + 4y - \pi = 0$ »

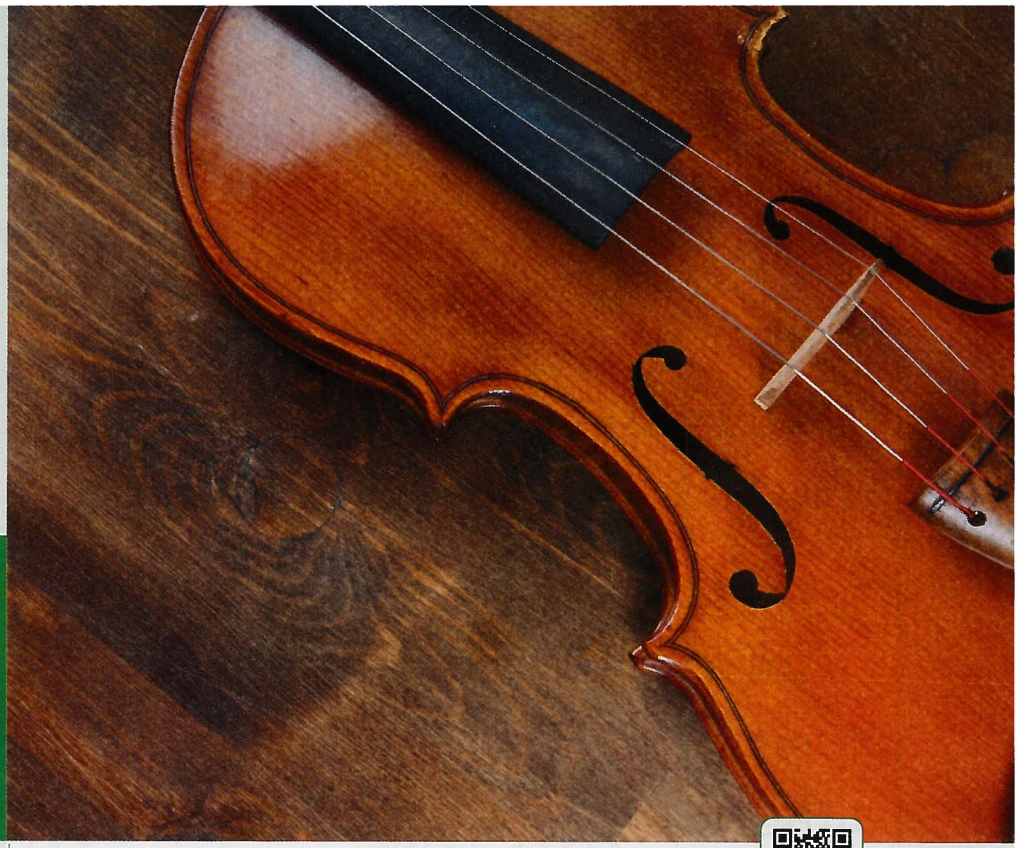
8 Find the points on the curve $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x + 1$ at which the tangent and the normal to the curve make an isosceles triangle with the x-axis.

9 If the tangent of the curve $y = x^2$ passes through the point $(3 , 5)$, then find the equation of this tangent. « $y - 2x + 1 = 0$ or $y - 10x + 25 = 0$ »

Exercise

15

Integration



From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from those given :

Integration of some algebraic functions

(1) $\int x \, dx =$ ..... $+ C$

(a) x^2 (b) $\frac{1}{2} x$ (c) $\frac{1}{2} x^2$ (d) 1

(2) $\int y^8 \, dy =$ ..... $+ C$

(a) $8 y^7$ (b) $\frac{1}{9} y^9$ (c) zero (d) y^9

(3) $\int (x + 2) \, dx =$ ..... $+ C$

(a) $2 x^2 + 2 x$ (b) $x^2 + 2 x$ (c) $\frac{1}{2} x^2 + 2 x$ (d) $(x + 2)^2$

(4) $\int (-3) \, dx =$ ..... $+ C$

(a) $-3 x$ (b) x (c) $\frac{(-3)^2}{2}$ (d) $\frac{-3}{2} x$

(5) The antiderivative of the function $f : f(x) = 3 x^2 - 2 x + 5$ is .....

(a) $6 x - 2$ (b) $3 x^2 - 2 x + 3$

(c) $x^3 - x^2 + 5 x + C$ (d) $\frac{1}{3} x^3 - \frac{1}{2} x^2 + 5 x + C$

(6) $\int 6 x^{-3} \, dx =$ ..... $+ C$

(a) $-3 x$ (b) $-3 x^{-1}$ (c) $-3 x^{-2}$ (d) $-18 x^{-4}$

(7) $\int x^{\frac{2}{3}} \, dx =$ ..... $+ C$

(a) $\frac{3}{5} x^{\frac{5}{3}}$ (b) $\frac{5}{3} x^{\frac{5}{3}}$ (c) $\frac{2}{3} x^{-\frac{1}{3}}$ (d) $x^{\frac{5}{3}}$

(8) $\int \frac{dx}{5x^4} =$ ..... + C

(a) $\frac{5}{x^5}$ (b) $\frac{-1}{5x^3}$ (c) $\frac{-1}{15x^3}$ (d) $5x^3$

(9) $\int \sqrt[7]{x^5}\, dx =$ ..... + C

(a) $\frac{12}{7} x^{\frac{12}{7}}$ (b) $\sqrt[7]{\frac{x^6}{6}}$

(c) $\frac{1}{6} x^6$ (d) $\frac{7}{12} x^{\frac{12}{7}}$

(10) $\int 5\sqrt{x^3}\, dx =$ ..... + C

(a) $2\sqrt{x^5}$ (b) $\frac{5}{2}\sqrt{x^4}$ (c) $5x^{\frac{5}{2}}$ (d) $\frac{5}{2}x^2$

(11) $\int \frac{x}{\sqrt{x}}\, dx =$ ..... + C

(a) $\frac{1}{2} x^{\frac{1}{2}}$ (b) $\frac{2}{3} x^{\frac{3}{2}}$ (c) $2x^{-\frac{1}{2}}$ (d) $\frac{2}{3} x^{-\frac{3}{4}}$

(12) $\int \left(\frac{2}{y^3} - \frac{4}{y^5}\right) dy =$ ..... + C

(a) $-y^2 + y^4$ (b) $-y^{-2} + y^{-4}$

(c) $\frac{2}{3} y^{-3} - \frac{4}{5} y^{-5}$ (d) $-2y^{-2} + 4y^{-4}$

(13) $\int (x^2 - 3x - 1)\, dx =$ ..... + C

(a) $\frac{1}{3} x^3 + 3x - x$ (b) $\frac{1}{3} x^3 - \frac{3}{2} x^2 - x$

(c) $x^3 - 3x^2 - x$ (d) $\frac{1}{2} (x^2 - 3x - 1)^2$

(14) $\int 2(3z^2 - 5)\, dz =$ ..... + C

(a) $(3z^2 - 5)^2$ (b) $6z^2 - 10$ (c) $2z^3 - 10z$ (d) $z^2 - 5z$

(15) $\int \left(2\sqrt{x} - 6x^2\right) dx =$ ..... + C

(a) $\frac{4}{3} x^{\frac{3}{2}} - 2x^3$ (b) $\frac{4}{3} x^{3\frac{1}{2}} - 2x^3$ (c) $x^{\frac{2}{3}} - 3x$ (d) $x^{\frac{-1}{2}} - 12x$

(16) $\int \left(3 - \frac{1}{\sqrt{x}}\right) dx =$ ..... + C

(a) $3x - \frac{1}{2} x^{\frac{1}{2}}$ (b) $\frac{3}{2} x^2 - \frac{1}{x}$ (c) $3x - \frac{1}{2x}$ (d) $3x - 2x^{\frac{1}{2}}$

(17) $\int x(x + 3)\, dx =$ ..... + C

(a) $\frac{1}{2} (x + 3)^2$ (b) $\frac{1}{3} x^3 + \frac{3}{2} x^2$ (c) $x^2 + 3x$ (d) $x^3 + 3x$

(18) $\int (x-5)(x+1)\,dx =$ ..... $+ C$

(a) $\frac{1}{3}x^3 - 2x^2 - 5x$ (b) $x^3 - 2x^2 - 5x$

(c) $\frac{2}{3}x^3 - 2x^2 + 5x$ (d) $\frac{1}{3}x^3 - 2x^2 + 5x$

(19) $\int (x+2)(x-2)\,dx =$ .....

(a) $x + 4 + C$ (b) $\frac{1}{3}x^3 - 4x + C$ (c) $x^3 - 4x + C$ (d) $(x^2-4)^2 + C$

(20) $\int \frac{x^2+3x}{x}\,dx =$ .....

(a) $x + 3$ (b) $\frac{1}{2}x^2 + 3x + C$ (c) $x^2 + 3x + C$ (d) $\frac{x^3+3x^2}{x^2}$

(21) $\int (x^2-2)^2\,dx =$ ..... $+ C$

(a) $\frac{1}{3}(x^2-2)^3$ (b) $2(x^2-2)(2x)$

(c) $\frac{1}{5}x^5 - \frac{4}{3}x^3 + 4x$ (d) $x^4 - 4x^2 + 4$

(22) $\int \left(\sqrt{x}-1\right)^2 dx =$ ..... $+ C$

(a) $\frac{1}{3}\left(\sqrt{x}-1\right)$ (b) $\frac{1}{2}x^2 - \frac{4}{3}x^{\frac{3}{2}} + x$

(c) $\frac{1}{3}\left(\sqrt{x}-1\right)^3$ (d) $\frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}}$

(23) $\int \left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)dx =$ ..... $+ C$

(a) $x^5 + x^{-3}$ (b) $x^4 - \frac{1}{x^4}$ (c) $\frac{1}{5}x^5 + \frac{1}{3}x^{-3}$ (d) $x^5 + \frac{1}{3}x^{-3}$

(24) $\int \frac{2x^2+3}{x^2}\,dx =$ ..... $+ C$

(a) $2x - 3$ (b) $\frac{1}{2}x^2 - 3x^{-1}$ (c) $x^2 - 3x$ (d) $2x - 3x^{-1}$

(25) $\int \frac{x^2-1}{x-1}\,dx =$ ..... $+ C$

(a) $x + 1$ (b) $\frac{1}{2}x^2 + x$ (c) $x^2 + x$ (d) $x^2 + 2x$

(26) $\int \frac{x^3-x}{x^2+x}\,dx =$ ..... $+ C$

(a) $\frac{1}{2}x^2 - x$ (b) $x^2 - x$ (c) $\frac{3}{2}x^3 - \frac{1}{2}x$ (d) $2x^2 + 1$

(27) $\int \frac{x^3+8}{x^2-2x+4}\,dx =$ ..... $+ C$

(a) $\frac{1}{4}(x+2)^4$ (b) $x + 2$ (c) $\frac{1}{2}x^2 + 2x$ (d) $\frac{1}{3}(x+2)^3$

(28) $\int (8-3x)^4 \, dx =$ ..... $+ C$

(a) $\frac{1}{15}(8-3x)^5$ (b) $\frac{1}{5}(8-3x)^5$ (c) $-\frac{1}{5}(8-3x)^5$ (d) $\frac{-1}{15}(8-3x)^5$

(29) $\int (2x+1)^5 \, dx =$ ..... $+ C$

(a) $(2x+1)^6$ (b) $\frac{1}{2}(2x+1)^6$

(c) $\frac{1}{12}(2x+1)^6$ (d) $\frac{1}{6}(2x+1)^6$

(30) $\int f(x) \cdot f'(x) \, dx =$ ..... $+ C$

(a) $f(x)$ (b) $\frac{1}{2}[f(x)]^2$ (c) $\frac{1}{2} f(x)$ (d) $[f(x)]^2$

(31) $\int (x-2)(x^2+2x+4) \, dx =$ ..... $+ C$

(a) $\frac{1}{4}x^4 - 4x^3 - 8x$ (b) $\frac{1}{3}x^3 + x^2 + 4x$

(c) $\frac{1}{4}x^4 - 8x$ (d) $x^3 - 8$

(32) $\int (x-1)(x^2-2x+1) \, dx =$ ..... $+ C$

(a) $\frac{1}{4}(x-1)^4$ (b) $\frac{1}{3}x^3 - x^2 + x$

(c) $\frac{1}{4}x^4 - x$ (d) $x^4 - 4x$

(33) $\int \frac{12}{(2x-5)^4} \, dx =$ ..... $+ C$

(a) $\frac{-1}{(2x-5)^3}$ (b) $\frac{6}{(2x-5)^3}$ (c) $\frac{2}{(2x-5)^3}$ (d) $\frac{-2}{(2x-5)^3}$

(34) $\int x^5 \left(1+\frac{3}{x}\right)^5 dx =$ ..... $+ C$

(a) $(x+3)^5$ (b) $\frac{1}{6}(x+3)^6$

(c) $\frac{1}{6}\left(1+\frac{3}{x}\right)^6$ (d) $\frac{1}{36}x^6\left(1+\frac{3}{x}\right)^6$

(35) $\int x \sqrt[5]{\frac{6}{x^4}+\frac{3}{x^5}} \, dx =$ ..... $+ C$

(a) $\frac{5}{36}(6x+3)^{\frac{6}{5}}$ (b) $\sqrt{6x+3}$ (c) $4(6x+3)^{\frac{3}{2}}$ (d) $\frac{7}{36}(6x+3)^{\frac{5}{6}}$

(36) $\int 4x(2x^2-1)^3 \, dx =$ ..... $+ C$

(a) $\frac{1}{2}x^2(2x^2-1)^4$ (b) $\frac{1}{4}(2x^2-1)^4$

(c) $x^2(2x^2-1)^4$ (d) $\frac{1}{4}(2x-1)^4$

(37) $\int (3x^2+1)(x^3+x+8)^5\,dx =$ ..... $+ C$

(a) $\frac{1}{3}(x^3+x+8)^6$ (b) $(3x^2+1)^2(x^3+x+8)^6$

(c) $\frac{1}{6}(x^3+x+8)^6$ (d) $\frac{1}{5}(x^3+x+8)^6$

(38) $\int x^2(2x^3+8)^4\,dx =$ ..... $+ C$

(a) $\frac{1}{30}(2x^3+8)^5$ (b) $\frac{1}{6}(2x^3+8)^5$

(c) $(2x^3+8)^5$ (d) $\frac{1}{20}x^4(2x^3+8)^5$

(39) $\int \frac{x-\frac{1}{2}}{\sqrt{2x-1}}\,dx =$ ..... $+ C$

(a) $\frac{1}{6}\sqrt{2x-1}$ (b) $\frac{1}{6}\sqrt{(2x-1)^3}$ (c) $\frac{1}{6\sqrt{2x-1}}$ (d) $\frac{1}{2}\sqrt{(2x-1)^3}$

(40) $\int \frac{d}{dx}[f(x)]\,dx =$ .....

(a) $f(x)$ (b) $f(x)+C$ (c) $\frac{d}{dx}[f(x)]$ (d) $\int f(x)\,dx$

(41) If $\int x^k\,dx = \frac{1}{3}x^3 + C$, then k = ...

(a) -1 (b) 1 (c) 2 (d) 3

(42) If $\int x^3\,dx = a\,x^4 + C$, then a = ...

(a) 4 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(43) If $\int x\sqrt{x^2-15}\,dx = a\sqrt{(x^2-15)^3} + C$, then a = ...

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

(44) $\int (x^2+4x+4)^5\,dx =$ ..... $+ C$

(a) $\frac{1}{6}(x^2+4x+4)^6$ (b) $6(x^2+4x+4)^5$

(c) $\frac{1}{11}(x+2)^{11}$ (d) $11(x+2)^{11}$

(45) $\int x(x+3)^5\,dx =$ ..... $+ C$

(a) $\frac{1}{6}(x+3)^6$ (b) $\frac{1}{18}(x+3)^6$

(c) $\frac{1}{7}(x+3)^7 - \frac{1}{2}(x+3)^6$ (d) $\frac{1}{6}(x+3)^6 - \frac{1}{5}(x+3)^5$

(46) $\int (x-1)(x+4)^8\,dx =$ ..... $+ C$

(a) $\frac{1}{10}(x+4)^{10} - \frac{5}{9}(x+4)^9$ (b) $\frac{-1}{10}(x+4)^9 - \frac{3}{4}(x+4)^8$

(c) $\frac{1}{2}(x-1)^2 + \frac{1}{9}(x+4)^9$ (d) $(x+4)^9 - 5(x+4)^3$

(47) $\int x\sqrt{x-2}\,dx =$ ..... $+ C$

(a) $\frac{3}{2}x^2(x-2)^{\frac{3}{2}}$

(b) $\frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{\frac{3}{2}}$

(c) $\frac{5}{2}(x-2)^{\frac{5}{2}} + \frac{3}{4}(x-2)^{\frac{3}{2}}$

(d) $\frac{2}{3}x^2(x-2)^{\frac{2}{3}}$

(48) $\int \frac{x+3}{\sqrt{x-1}}\,dx =$ ..... $+ C$

(a) $\frac{2}{3}(x-1)^{\frac{3}{2}} + 8(x-1)^{\frac{1}{2}}$

(b) $\frac{1}{2}(x+3)^2 - \frac{3}{2}(x-1)^{\frac{-3}{2}}$

(c) $\frac{-2}{3}(x-1)^{\frac{3}{2}} + 8(x-1)^{\frac{1}{2}}$

(d) $\frac{3}{2}(x-1)^{\frac{3}{2}} + 8(x-1)^{\frac{1}{2}}$

Integration of some trigonometric functions

(49) $\int \cos(4x)\,dx =$ ..... $+ C$

(a) $\frac{1}{4}\sin(4x)$

(b) $\frac{1}{2}\cos(2x)$

(c) $\frac{1}{4}\cos^2(4x)$

(d) $4\sin(4x)$

(50) $\int \sin(3x)\,dx =$ ..... $+ C$

(a) $\frac{1}{3}\sec^2(3x)$

(b) $-\frac{1}{3}\cos(3x)$

(c) $\frac{1}{3}\cos(3x)$

(d) $\frac{1}{3}\sin(3x)$

(51) $\int \sec^2(5x)\,dx =$ ..... $+ C$

(a) $\frac{1}{15}\sec^3(5x)$

(b) $\frac{1}{2}\tan^2(2x)$

(c) $\frac{1}{5}\tan(5x)$

(d) $\frac{1}{5}\cot(5x)$

(52) $\int (\sin^2 x + \cos^2 x)^7\,dx =$ ..... $+ C$

(a) x

(b) $2\sin 2x$

(c) $\frac{1}{8}x^8$

(d) $\frac{1}{3}\sin^3 x + \frac{1}{3}\cos^3 x$

(53) $\int \sin(3x-5)\,dx =$ ..... $+ C$

(a) $-\cos(3x-5)$

(b) $-3\cos(3x-5)$

(c) $\frac{-1}{3}\cos(3x-5)$

(d) $\frac{-2}{3}\cos(3x-5)$

(54) $\int (x - \sin x)\,dx =$ ..... $+ C$

(a) $3\sin\left(\frac{x}{3} - 2\right)$

(b) $\frac{1}{2}x^2 + \cos x$

(c) $\frac{1}{2}x^2 - \cos x$

(d) $1 + \cos x$

(55) $\int \sin\left(\frac{x}{4}+\frac{\pi}{3}\right) dx = \ldots\ldots\ldots + C$

(a) $-4\cos\left(\frac{x}{4}+\frac{\pi}{3}\right)$ (b) $\cos\left(\frac{x}{4}+\frac{\pi}{3}\right)$

(c) $\frac{1}{2}\cos\left(\frac{x}{4}+\frac{\pi}{3}\right)$ (d) $-4\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)$

(56) $\int \cos\left(\frac{\pi}{2}+x\right) dx = \ldots\ldots\ldots + C$

(a) $\sin x$ (b) $\cos x$ (c) $-\sin x$ (d) $-\cos x$

(57) $\int 2\sin x \cos x\, dx = \ldots\ldots\ldots + C$

(a) $\frac{1}{2}\sin(2x)$ (b) $-\frac{1}{2}\cos(2x)$

(c) $-\cos(2x)$ (d) $\sin(2x)$

(58) $\int \sec^2\left(\frac{\pi}{4}\right) dx = \ldots\ldots\ldots + C$

(a) x (b) $2x$ (c) $\tan x$ (d) $\tan\frac{\pi}{4}$

(59) $\int (\sin x \csc x + \cos x \sec x + \tan x \cot x)\, dx = \ldots\ldots\ldots + C$

(a) 3 (b) $3x$

(c) $\cos x - \sin x + \sec^2 x$ (d) $\frac{1}{2}\sin^2 x + \frac{1}{2}\cos^2 x + \frac{1}{2}\sec^2 x$

(60) If $\int \sin x\, dx = f(x)$, then $\dot{f}(x) = \ldots\ldots\ldots$

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$

(61) $\int (\cos^2 x - \sin^2 x)\, dx = \ldots\ldots\ldots + C$

(a) $\frac{1}{3}\cos^3 x - \frac{1}{3}\sin^3 x$ (b) $\cos x$

(c) $\frac{1}{2}\sin 2x$ (d) $2\sin x$

(62) $\int \sin 3x \cos 3x\, dx = \ldots\ldots\ldots + C$

(a) $\sin 3x$ (b) $\frac{1}{3}\sin 6x$ (c) $\frac{1}{2}\sin^2 3x$ (d) $\frac{-1}{12}\cos 6x$

(63) If $\int \cos(3x+1)\, dx = a\sin(3x+1) + C$, then $a = \ldots\ldots\ldots$

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{9}$

(64) $\int \left(\frac{\tan x}{\cot x}+1\right) dx = \ldots\ldots\ldots + C$

(a) $\cot x$ (b) $-\tan x$ (c) $\tan x$ (d) $-\cot x$

(65) $\int (2+\tan^2 x)\, dx = \ldots\ldots\ldots + C$

(a) $x - \tan x$ (b) $x + \cot x$ (c) $-x + \tan x$ (d) $x + \tan x$

(66) $\int \cot x \sin x \, dx =$ ..... $+ C$

(a) $-\sin x$ (b) $\sin x$ (c) $\cos x$ (d) $-\cos x$

(67) $\int (\sin x + \cos x)^2 \, dx =$ ..... $+ C$

(a) $x - \frac{1}{2}\cos 2x$ (b) $x + \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$

(c) $x + \sin^2 x$ (d) $x - \cos^2 x$

(68) $\int (\sin^2 5x + \cos^2 5x + \tan^2 5x) \, dx =$ ..... $+ C$

(a) $\frac{1}{5}\sec^2 5x$ (b) $x + \frac{1}{5}\tan 5x$ (c) $1 + \frac{1}{5}\sec^2 5x$ (d) $\frac{1}{5}\tan 5x$

(69) $\int (1 + \tan^2 x)\cos^3 x \, dx =$ ..... $+ C$

(a) $\cos x$ (b) $\sin x$ (c) $-\cos x$ (d) $-\sin x$

(70) $\int \frac{\sin x + \cos x}{1 + \cot x} \, dx =$ ..... $+ C$

(a) $\cos x$ (b) $-\cos x$ (c) $\sin x$ (d) $-\sin x$

(71) $\int \frac{1 - \cos^2 x}{\sin x} \, dx =$ ..... $+ C$

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$

(72) $\int \frac{d}{dx}(5 + \sin x) \, dx =$ .....

(a) $5x - \cos x$ (b) $5x + \sin x + C$ (c) $\sin x + C$ (d) $5x - \cos x + C$

(73) $\int \frac{5 + \sec x}{\sec x} \, dx =$ ..... $+ C$

(a) $5\sin x - x$ (b) $-5\sin x - x$ (c) $2\sin x - x$ (d) $5\sin x + x$

(74) $\int \frac{\cos^2 x + 2}{\cos^2 x} \, dx =$ ..... $+ C$

(a) $x + \tan x$ (b) $2x + \tan x$ (c) $x + 2\tan x$ (d) $x - 2\cot x$

(75) $\int \tan^9 x \cot^9 x \, dx =$ ..... $+ C$

(a) $-x + \tan x$ (b) $x + \tan x$ (c) x (d) $-x + \cot x$

(76) $\int \frac{1}{\cos^2 x} \, dx =$ ..... $+ C$

(a) $\cot x$ (b) $\tan x$ (c) $\sec^2 x$ (d) $\csc^2 x$

(77) $\int \sin^2 x \, dx =$ ..... $+ C$

(a) $\frac{1}{2}x - \frac{1}{4}\sin 2x$ (b) $x - \frac{1}{2}\sin 2x$

(c) $\frac{1}{4}\cos 2x + \frac{1}{2}x$ (d) $\frac{1}{2}x - \frac{1}{2}\sin 2x$

(78) $\int \frac{1+\cos^2 x}{1-\sin^2 x}\,dx =$ ..... $+ C$

(a) $x + \cos x$ (b) $\tan x + x$ (c) $\sec^2 x + x$ (d) $x - \sec^2 x$

(79) $\int \frac{\cos 2x}{\cos x + \sin x}\,dx =$ ..... $+ C$

(a) $\sin 2x + \cos 2x$ (b) $2 \sin x \cos x$

(c) $\cos x - \sin x$ (d) $\cos x + \sin x$

(80) $\int \frac{dx}{1+\tan^2 x} =$ ..... $+ C$

(a) $\cos^2 x$ (b) $\frac{1}{4}\sin 2x + \frac{1}{2}x$ (c) $\tan x + 1$ (d) $\frac{1}{2}\cos^2 x$

(81) $\int \frac{4}{1+\cos 4x}\,dx =$ ..... $+ C$

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$ (d) $\tan x$

(82) $\int \frac{\tan^3 x}{\cos^2 x}\,dx =$ ..... $+ C$

(a) $\frac{1}{3}\tan^3 x$ (b) $\frac{1}{4}\tan^4 x$ (c) $\frac{1}{3}\cos^3 x$ (d) $\frac{1}{4}\sin^4 x$

(83) $\int \frac{\sin x\,(1+\cos 2x)}{\sin 2x}\,dx =$ ..... $+ C$

(a) $\sin x$ (b) $\cos x$ (c) $\sin 2x$ (d) $\cos 2x$

(84) $\int \frac{dx}{(\cos^2 x - \sin^2 x)^2} =$ ..... $+ C$

(a) $-(\cos^2 x - \sin^2 x)^{-1}$ (b) $\sec^2 2x$

(c) $\frac{1}{2}\tan 2x$ (d) $\frac{1}{2}\sec^2 2x$

(85) $\int \sin x \cos x \cos 2x\,dx =$ ..... $+ C$

(a) $-\frac{1}{16}\sin 4x$ (b) $\frac{-1}{4}\cos 4x$ (c) $\frac{-1}{16}\cos 4x$ (d) $\frac{-1}{4}\sin 4x$

(86) $\int (1+\sec x)(1-\sec x)\,dx =$ ..... $+ C$

(a) $x - \tan x$ (b) $x + \tan x$ (c) $x \tan x$ (d) $x - \cot x$

(87) $\int (1-\cos x)(1+\cos x)\,dx =$ ..... $+ C$

(a) $\sin^2 x$ (b) $1 + \cos x$

(c) $\frac{1}{2}x - \frac{1}{4}\sin 2x$ (d) $x + \frac{1}{2}\sin 2x$

(88) $\int \sin^{100} x \cdot \csc^{99} x\,dx =$ ..... $+ C$

(a) $\sin x$ (b) $-\cos x$ (c) $\sin x \tan x$ (d) $\frac{1}{101}\cos^{101} x$

(89) $\int \frac{\cos x}{\csc x} \, dx =$ ..... $+ C$

(a) $\frac{1}{2} \sin 2x$ (b) $\frac{-1}{4} \cos 2x$ (c) $2 \sin 2x$ (d) x

(90) $\int \sqrt{1 - \cos^2 x} \, dx =$ ..... $+ C$ where $x \in]0 , \pi[$

(a) $\sin x$ (b) $\cos x$ (c) $x - \cos x$ (d) $- \cos x$

(91) $\int \left(\cos^4 \frac{x}{2} - \sin^4 \frac{x}{2}\right) dx =$ ..... $+ C$

(a) $\cos x$ (b) $\cos 2x$ (c) $\sin x$ (d) $\sin 2x$

(92) $\int \sin^3 x \, dx =$ ..... $+ C$

(a) $\cos^3 x - \frac{1}{3} \cos^3 x$ (b) $\cos x + \frac{1}{3} \cos^3 x$

(c) $- \cos x + \frac{1}{3} \cos^3 x$ (d) $- \sin x - \frac{1}{3} \sin^3 x$

(93) $\int (\sin x + \tan x)^5 (\cos x + \sec^2 x) \, dx =$ ..... $+ C$

(a) $\frac{1}{5} (\sin x + \tan x)^5$ (b) $\frac{1}{6} (\sin x + \tan x)^6$

(c) $\frac{1}{6} (\sin^6 x + \tan^6 x)$ (d) $\frac{1}{6} \cos^5 2x + x$

(94) $\int (\sin 2x \sin x + \cos 2x \cos x) \, dx =$ ..... $+ C$

(a) $2 \cos x$ (b) $- \cos x$ (c) $- 2 \sin x$ (d) $\sin x$

Second Essay questions

Integration of some algebraic functions

1 **Find each of the following integrations :**

(1) $\int \frac{dx}{5}$ | (2) $\int (\text{zero}) \, dx$ | (3) $\int \frac{-8}{5} s^{-3} \, ds$

(4) $\int \frac{1}{\sqrt[5]{x^3}} \, dx$ | (5) $\int \frac{12}{x\sqrt{x}} \, dx$ | (6) $\int \frac{6\sqrt{y}}{\sqrt[3]{y}} \, dy$

2 **Find each of the following integrations :**

(1) $\int (4x^3 - 6x^2 + 8x - 5) \, dx$ | (2) $\int (x^4 - x^{-2} + 5) \, dx$

(3) $\int \left(\frac{3}{x^3} + \frac{2}{x^2} + 5\right) dx$ | (4) $\int \left(3\sqrt{x} - \frac{2}{x^3}\right) dx$

(5) $\int (ax^2 + 5x + c) \, dx$ | (6) $\int \left(\frac{5}{3}\sqrt[3]{x^2} + \frac{7}{4}\sqrt[4]{x^3}\right) dx$

3 **Find each of the following integrations :**

(1) $\int x^2\left(3x-\frac{5}{x}\right)dx$

(2) $\int (x-2)(4x^2+1)\,dx$

(3) $\int x^3(2-x)^2\,dx$

(4) $\int (1-x^2)^2(1+x^2)^2\,dx$

(5) $\int \left(x+\frac{1}{x}\right)^2 dx$

(6) $\int \sqrt{x}\left(1+\sqrt{x}\right)dx$

(7) $\int \sqrt{x}\left(x\sqrt{x}+2\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$

4 **Find each of the following integrations :**

(1) $\int \frac{x^7-4x^5+x^3}{x^3}dx$

(2) $\int \frac{x^2-5x+6}{x-2}dx$

(3) $\int \frac{x-4}{\sqrt{x}+2}dx$

(4) $\int \frac{x^3-27}{x-3}dx$

(5) $\int \frac{4x^3-12x^2}{x-3}dx$

(6) $\int \frac{(x^2+2)^2}{x^2}dx$

(7) $\int \frac{x^3+x-2}{x-1}dx$

(8) $\int \frac{x^2-4}{\sqrt{x}(x-2)}dx$

5 **Find each of the following integrations :**

(1) $\int 6(x-2)^5\,dx$

(2) $\int 3(3x+1)^3\,dx$

(3) $\int (2z-3)^{\frac{7}{3}}\,dz$

(4) $\int \sqrt{2x+7}\,dx$

(5) $\int (3x+5)^{-9}\,dx$

(6) $\int \left((3-2x)^5+3\right)dx$

(7) $\int \frac{dx}{\sqrt[5]{2x+9}}$

(8) $\int \sqrt[3]{(4-3x)^2}\,dx$

6 **Find each of the following integrations :**

(1) $\int x^6\left(1-\frac{2}{x}\right)^6 dx$

(2) $\int x\sqrt[3]{\frac{2}{x^3}-\frac{1}{x^2}}\,dx$

(3) $\int x^{10}\left(\frac{2}{x}+\frac{3}{x^2}\right)^5 dx$

(4) $\int (4x^2-12x+9)^{\frac{7}{2}}dx$

(5) $\int (4x+6)\sqrt{2x+3}\,dx$

(6) $\int \sqrt{x-3}\,(x-3)^4\,dx$

(7) $\int (x+1)\sqrt[3]{x^2+2x+1}\,dx$

(8) $\int \frac{8-2x}{\sqrt{4-x}}dx$

7 **Find each of the following integrations :**

(1) $\int x(5x^2+2)^{-7}\,dx$

(2) $\int (x^2-3x+5)^{-7}(2x-3)\,dx$

(3) $\int (3x^2-2x+1)^{11}(3x-1)\,dx$

(4) $\int x(4x^3-3x^2+4)^5(2x-1)\,dx$

(5) $\int x^2(x^4-x^3+8)^9(4x-3)\,dx$

(6) $\int \frac{x}{\sqrt{1+x^2}}\,dx$

(7) $\int \frac{x^2+1}{(x^3+3x)^2}\,dx$

(8) $\int \frac{x^3}{(x^4+1)^6}\,dx$

8 **Find each of the following integrations :**

(1) $\int (x+1)(x+3)^7\,dx$

(2) $\int 3x\sqrt[3]{3x+1}\,dx$

(3) $\int (x+2)\sqrt{x+1}\,dx$

(4) $\int x^{17}\left(\frac{1}{x}-\frac{1}{x^2}\right)^8 dx$

(5) $\int \frac{x+3}{(x-2)^4}\,dx$

(6) $\int \frac{x+1}{(3x+1)^5}\,dx$

(7) $\int \frac{3x+2}{\sqrt{x-2}}\,dx$

(8) $\int \frac{x^2+8x}{(x+4)^5}\,dx$

(9) $\int \frac{x^2+3x+2}{(x+2)^5}\,dx$

(10) $\int \sqrt{x^3-x^2}\,dx$ where $x>0$

Integration of some trigonometric functions

9 **Find each of the following integrations :**

(1) $\int \cos\left(\frac{x}{3}-2\right)dx$

(2) $\int \sec^2\left(\frac{x}{3}-1\right)dx$

(3) $\int (6x+\cos 4x)\,dx$

(4) $\int \cos(2-x)\,dx$

(5) $\int \left(4x-\sin\frac{\pi}{3}\right)dx$

(6) $\int \left(\frac{\pi}{2}-2\sin\frac{3x}{2}\right)dx$

(7) $\int (\cos 3x-\sin x)\,dx$

(8) $\int (7\sin x-2\cos x)\,dx$

(9) $\int 9(\cos x-\sec^2 3x)\,dx$

(10) $\int (3+5\sec^2 x+2\sin 2x)\,dx$

(11) $\int \left(4\cos x+\frac{1}{\cos^2 x}+1\right)dx$

(12) $\int \left[\sec^2\frac{x}{2}-\sin\left(\frac{\pi}{4}-x\right)\right]dx$

10 **Find each of the following integrations :**

(1) $\int (\sin^2 3x+\cos^2 3x+\tan^2 3x)\,dx$

(2) $\int (1-2\sin^2 3x)\,dx$

(3) $\int \sin x\cos x\,dx$

(4) $\int \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2 dx$

(5) $\int \left(1+\tan^2\frac{x}{2}\right) dx$

(6) $\int (3+4\tan^2 x)\, dx$

(7) $\int \frac{\cos^2 x}{1+\sin x}\, dx$

(8) $\int \frac{1+\sin^2 x}{1-\sin^2 x}\, dx$

(9) $\int \frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}\, dx$

(10) $\int \left[(1-\tan x)^2+2\tan x\right] dx$

(11) $\int \left(\frac{\cos^2 x+1}{\cos x}\right)^2 dx$

(12) $\int \frac{3}{\cos^2 3x}\, dx$

(13) $\int \sec x\,(3\cos x-2\sec x)\, dx$

(14) $\int (\sin 3x\cos x-\cos 3x\sin x)\, dx$

(15) $\int \left(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}\right) dx$

11 Find each of the following integrations :

(1) $\int 2\cos^2 x\, dx$

(2) $\int (1+\cos x)^2\, dx$

(3) $\int (1-\sin x)^2\, dx$

(4) $\int (1+\sin^2 x)\, dx$

(5) $\int (4-\sin^2 x)\, dx$

(6) $\int (2\cos^2 x+\tan^2 x)\, dx$

(7) $\int (\sec x+\cos x)^2\, dx$

(8) $\int \left(\frac{2}{\csc^2 x}-3\sec^2 2x\right) dx$

12 Find each of the following integrations :

(1) $\int \sin^5 x\cos x\, dx$

(2) $\int 2\tan x\sec^2 x\, dx$

(3) $\int (2x-\sin x)(x^2+\cos x)^7\, dx$

(4) $\int 13\tan^{12} x\sec^2 x\, dx$

(5) $\int \tan^2 x\csc^2 x\, dx$

(6) $\int \sin x\cos x\cos 2x\cos 4x\, dx$

(7) $\int (1+\tan^2 y)\cos^2 y\, dy$

(8) $\int \cos x\,(\sin x+\tan x)\, dx$

(9) $\int \left(\sec^2\frac{\pi}{4}+\frac{1-2\sin^2 x}{\cot 2x}\right) dx$

(10) $\int (\sin^2 x+\sin^2 x\tan^2 x)\, dx$

(11) $\int \frac{\cos 4x}{\cos 2x-\sin 2x}\, dx$

(12) $\int \frac{2\tan^2 x}{1+\tan^2 x}\, dx$

(13) $\int \sqrt{1+\cos 2\theta}\, d\theta$ where $\theta\in\left]0,\frac{\pi}{4}\right[$

(14) $\int 8\sin^2\frac{x}{2}\cos^2\frac{x}{2}\, dx$

(15) $\int (\cos^2 x-\cos^4 x)\, dx$

(16) $\int (\cos x+\tan x)^2\, dx$

(17) $\int \frac{(\sin x+\cos x)^2}{\sec^2 x+2\tan x}\, dx$

(18) $\int 4\sin^4 x\, dx$

Third Higher skills

Integration of some algebraic functions

Choose the correct answer from those given :

(1) If $f(x) = \int \frac{1}{x} dx$, then $\grave{f}(2) =$ .....

(a) not exist. (b) $\frac{1}{x} + C$ (c) 2 (d) $\frac{1}{2}$

(2) $\int \frac{x^2}{x+2} dx + \int \frac{4x+4}{x+2} dx =$ ..... $+ C$

(a) $\frac{1}{2}x^2 + 2x$ (b) $-\frac{1}{2}x^2 + 2x$ (c) $x^2 + 4x$ (d) $x^2 + x$

(3) $\int x f(x) dx = x^4 - 8x^2$, then $f(1) =$ .....

(a) 12 (b) 24 (c) – 12 (d) – 24

(4) $\int [2x + 2f(x) - 2][1 + \grave{f}(x)] dx =$ ..... $+ C$

(a) $x + f(x) - 1$ (b) $(x + f(x) - 1)^2$

(c) $x^2 + (f(x))^2 - x$ (d) $\frac{1}{2}x^2 + \frac{1}{2}(f(x))^2 - 2x$

(5) If $\int f(x) dx = x^3 - 5x^2 + 7x + 2$, then $\grave{f}(1) =$ .....

(a) 5 (b) zero (c) – 3 (d) – 4

(6) $\int \sqrt{(x^2 - \frac{1}{x^2})^2 + 4} dx =$ ..... $+ C$

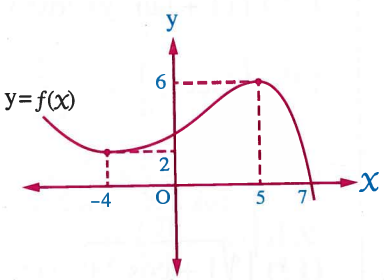
(a) $\frac{1}{3}x^3 - \frac{1}{x}$ (b) $(\frac{1}{3}x^3 + \frac{1}{x})^2$

(c) $\frac{1}{3}x^3 + \frac{1}{x}$ (d) $\frac{1}{3}(\frac{1}{3}x^3 - \frac{1}{x})^3$

(7) The opposite figure shows the curve $y = f(x)$ and

$g(x) = \int [f(x) + 1] . \grave{f}(x) dx$

, then $g(5) - g(-4) =$ .....



(a) 12 (b) 16

(c) 18 (d) 20

Integration of some trigonometric functions

(8) $\int \frac{\sin x}{\cos^3 x} dx =$ ..... $+ C$

(a) $\frac{1}{2}\tan^2 x$ (b) $\sec x$ (c) $2 \sec x$ (d) $\frac{1}{2}\tan x \sec x$

(9) $\int \frac{1+\cos x}{1+\sec x} dx = \ldots\ldots\ldots + C$

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$

(10) $\int \frac{\cos x + \cos^3 x}{\sin^2 x + 2\cos^2 x} dx = \ldots\ldots\ldots + C$

(a) $\cos x$ (b) $\sin x$ (c) $\frac{1}{2}\cos^2 x$ (d) $\frac{1}{2}\sin^2 x$

(11) $\int \frac{1-\cos x}{1+\cos x} dx = \ldots\ldots\ldots + C$

(a) $2\tan\frac{x}{2} - x$ (b) $\tan x - x$ (c) $2\tan^2\frac{x}{2} - x$ (d) $2\tan\frac{x}{2} - 1$

(12) If $a = \int \cos^2 x\, dx$, $b = \int \sin^2 x\, dx$, then $a + b = \ldots\ldots\ldots$

(a) zero (b) 1 (c) $x + C$ (d) $\frac{1}{2}\sin 2x + C$

(13) $\int (\tan^3 x + \tan x)\, dx = \ldots\ldots\ldots + C$

(a) $\frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x$ (b) $\frac{1}{2}\tan^2 x$

(c) $\frac{1}{2}(\tan^3 x + \tan x)^2$ (d) $\tan x \sec^2 x$

(14) $\int \frac{(\cot x - \csc x)(\cot x + \csc x)}{(\tan x - \sec x)(\tan x + \sec x)} dx = \ldots\ldots\ldots + C$

(a) x (b) $-x$ (c) $\frac{1}{2}x$ (d) $2x$

(15) $3\int \sin 2x \sin^4 x\, dx = \ldots\ldots\ldots$

(a) $\sin^4 x \cos^2 x + C$ (b) $\cos^4 x \sin^2 x + C$

(c) $\sin^6 x + C$ (d) $\cos^6 x + C$

Unit Four

Trigonometry

- Accumulative exercise on what we studied before.

Exercise 16 Angles of elevation and depression "Applications on solving the triangle"

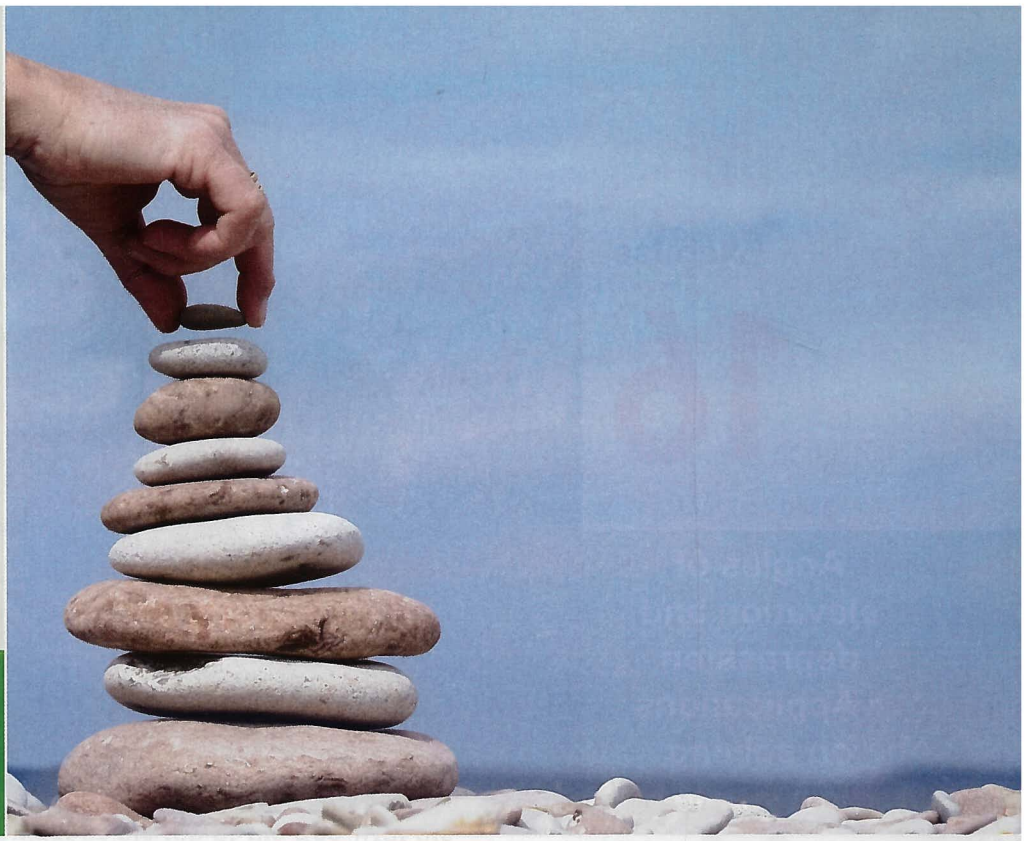
Exercise 17 Trigonometric functions of the sum and difference between the measures of two angles.

Exercise 18 Trigonometric functions of double of the measure of an angle.

Exercise 19 Heron's formula.

Scan the QR code to solve an interactive test on each lesson

Accumulative exercise on what we studied before



1 **Choose the correct answer from the given ones :**

(1) In Δ XYZ , 2r sin X = ...

(a) z (b) $\mathcal{X}$ (c) y (d) the area of Δ XYZ

(2) If $\angle$ A supplements $\angle$ C , then cos A + cos C = ...

(a) zero (b) 1 (c) – 1 (d) $\frac{1}{2}$

(3) In any triangle XYZ we find XY : YZ = ...

(a) sin X : sin Y (b) sin Y : sin Z

(c) sin Z : sin X (d) sin Z : sin Y

(4) ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

(5) In Δ XYZ , if $\mathcal{X}$ = y , then cos X = ...

(a) $\frac{2 y^2}{z}$ (b) $\frac{z}{2 y}$ (c) $\frac{z}{4 \mathcal{X}}$ (d) $\frac{y}{2 \mathcal{X}}$

---

2 XYZ is a triangle in which m ($\angle$ X) = 80° , m ($\angle$ Y) = 60° , z = 10 cm.

Find each of : $\mathcal{X}$, y to the nearest cm. « 15 cm. , 13 cm. »

---

3 Solve the triangle ABC in which m ($\angle$ A) = 50° , a = 4 cm. , b = 3 cm.

---

4 ABCD is a quadrilateral in which AB = AD = 9 cm. , BC = 5 cm. , CD = 8 cm. , AC = 11 cm. **Prove that :** ABCD is a cyclic quadrilateral.

Exercise

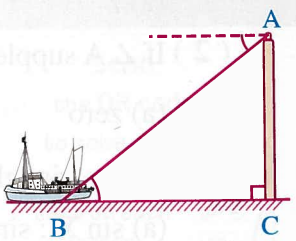
16

Angles of elevation and depression « Applications on solving the triangle »

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from those given :



(1) From the top of a lighthouse , a boat was observed at an angle of depression of measure 38° , if the horizontal distance between the boat and the base of the lighthouse is 220 m. , then the height of the lighthouse ≃ ..... m.

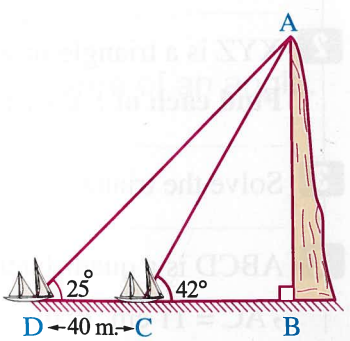
(a) 164 (b) 172 (c) 186 (d) 196

(2) From a point on the ground surface a man observed the elevation angle of the top of a building to be of measure θ , then the measure of the depression angle of the man position from the top of the building is .....

(a) θ (b) 90° – θ (c) θ – 90° (d) 180° – θ

(3) From a point on the ground , a child observed the top of a tower of height 50 m. , if the child was at a distance $50\sqrt{3}$ m. from the base of the tower , then the measure of the elevation angle of the top of the tower = ...

(a) 45° (b) 120° (c) 60° (d) 30°



(4) A man in a boat , observed the angle of elevation of the top of a rock of measure 25°. He moved horizontally towards the base of the rock 40 m. He then measured the elevation angle of the top of the rock and it was of measure 42° , then the height of the rock ≃ ..... m.

(a) 39 (b) 42 (c) 46 (d) 51

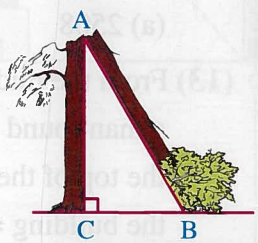
(5) From the top of a tower of height 65 m. , the measures of the two angles of depression of the two points A and B are 32° and 21° 13̀ respectively. If D represents the base of the tower and $A \in \overline{BD}$, then the length of $\overline{AB} \simeq$ ..... m.

(a) 54 (b) 58 (c) 63 (d) 67

(6) From a window of a building 8 m. high , the elevation and depression angles of the top and the bottom of a tree were measured and found to be equal , given that the bases of the building and the tree are on the same horizontal plane , then the height of the tree = ... m.

(a) 8 (b) $8\sqrt{2}$ (c) 16 (d) 24

(7) Because of the wind , the upper part of a tree is broken and makes an angle of measure 60° with the ground , if the point of contact of the top of the tree with the ground was at a distance 10 m. from its bottom , then the tree height $\simeq$ ..... m.



(a) 32 (b) 35 (c) 37 (d) 42

(8) As a man approche to the base of a tower , the measure of the elevation angle of the top of the tower .....

(a) is increasing. (b) is decreasing.

(c) remains constant. (d) cannot be determined.

(9) If a man measure the elevation angle of the top of a tower from point (A) on the horizontal plane passing through the base of the tower , it was θ_1 when the man ascends vertically up (S) m. and measures the elevation angle of the top of the tower again it was θ_2 , then .....

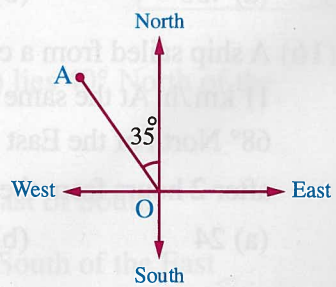
(a) $\theta_1 < \theta_2$ (b) $\theta_1 > \theta_2$

(c) $\theta_1 = \theta_2$ (d) $\theta_1 + \theta_2 = 90°$

(10) In the opposite figure :

The point (A) lies .....

with respect to the point (O)

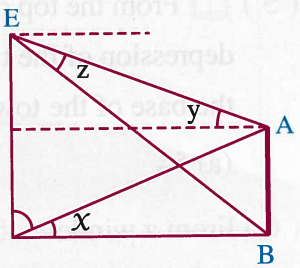


(a) North (b) West

(c) 35° North of the West (d) 35° West of the North

(11) In the opposite figure :

The measure of the elevation angle of the point (E) when it is measured from the point (B) equals .....



(a) z (b) y

(c) $\mathcal{X}$ (d) y + z

(12) From the top of a mountain , it was found that the measure of the depression angle of a car moving with constant velocity towards the mountain was 40° , after two minutes , the measure of the depression angle of the same car was 67° , then the height of the mountain , knowing that the velocity of the car was 60 km./h. equals ..... m.

(a) 2508 (b) 2607 (c) 2706 (d) 2807

(13) From the midpoint between two bases of a building and a tree of height 15 m. , a man found that the measures of the two angles of elevation of the top of the tree and the top of the building were 30° , 60° respectively , then the height of the building = ... m.

(a) 40 (b) $15\sqrt{3}$ (c) $30\sqrt{3}$ (d) 45

(14) An aeroplane C was observed from the two points A and B on the ground surface while passing through the vertical plane that passes through $\overleftrightarrow{AB}$ where AB = 3000 m. , then the measure of its elevation angle from A was 53° 21̀ and the measure of its elevation angle from B was 34° 26̀ and the projection of the aeroplane $\in \overline{AB}$, then the height of the aeroplane from the ground surface $\simeq$ ..... m.

(a) 1154 (b) 1262 (c) 1322 (d) 1362

(15) From an aeroplane , it was found that the measure of the angle of depression of a fixed point on the ground was 60° When the aeroplane came down at a distance 200 m. , it was found that the measure of the angle of depression of the same point became 45° , then the height of the aeroplane at the same moment of the first measurement $\simeq$ ..... m.

(a) 453 (b) 473 (c) 493 (d) 512

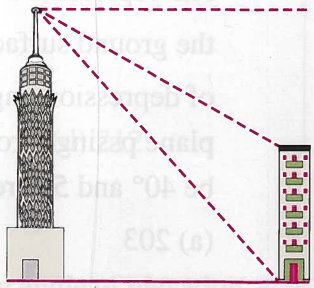
(16) A ship sailed from a certain point in the direction of 12° South of the East with velocity 11 km./h. At the same moment another ship sailed from the same point in the direction of 68° North of the East with velocity 6.5 km./h. , then the distance between the two ships after 2 hours from the time of their sailing together $\simeq$ ..... km.

(a) 24 (b) 26 (c) 28 (d) 30

(17) Three villages A , B and C. The village A lies at the West of B where AB = 20 km. , the village C lies in the direction 48° East of North of A and 60° North of West of B , then the distance between the villages B and C ≃ ..... km.

(a) 13 (b) 14 (c) 15 (d) 16

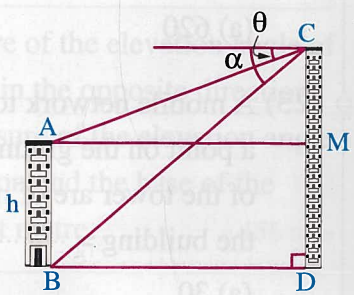
(18) From the top of cairo tower , a man used the scope to observe the angles of depression of the top and the base of his house , which is located near to the tower , were 45° , 60° respectively. If the height of the tower is 187 m. , then the height of the house ≃ ..... metre.



(a) 96 (b) 87 (c) 108 (d) 79

(19) In the opposite figure :

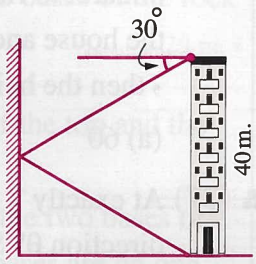
$\overline{CD}$ represents a tower , $\overline{AB}$ represents a house of height h metre and θ , α are the measures of the depression angles of A and B from C respectively , then the distance between the top of the tower and the top of the house = ... m.



(a) $\frac{h \sin \alpha}{\sin \theta}$ (b) h sin α cos θ

(c) $\frac{h \cos \theta}{\sin (\alpha - \theta)}$ (d) $\frac{h \cos \alpha}{\sin (\alpha - \theta)}$

(20) If you know that from the top of a building , the depression angle of the reflection of the base image of a building through avertical mirror placed on a tower opposite to the building is 30° and the building height = 40 m. , then the distance between the building and the tower is ..... metre.



(a) 40 (b) 30 (c) $20\sqrt{3}$ (d) 20

(21) If point (A) lies 30° North of the East of (O) and point (B) lies 40° North of the West of (O) , OA = OB , then (A) lies ..... of (B)

(a) 5° South of the East (b) 5° East of South

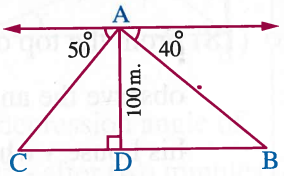
(c) 35° North of the West (d) 35° South of the East

(22) From the top of a tower , the depression angles of a minaret top and base are measured , their measures are $32°27\hat{}$, $57°52\hat{}$ respectively. If the height of the minaret is 37 m. then the height of the tower = ... m.

(a) 57 (b) 62 (c) 67 (d) 71

(23) **In the opposite figure :**

$\overline{AD}$ represents a light house of height 100 metre from the ground surface , a person from its top observed measure of depression angles of two boats "B , C" in the same horizontal plane pssing through the light house base and in two opposite sides from it to be 40° and 50° respectively , then distance between the two boats (BC) ≃ ..... m.



(a) 203 (b) 152 (c) 314 (d) 232

(24) A steam boat moves in a straight line towards a rock with a uniform velocity 300 m./min. At a certain moment , the elevation angle of the top of the rock is measured , the measure of the elevation angle was 35°. After two minutes it was measured again and it was 60° , then the height of the rock = ... m.

(a) 620 (b) 685 (c) 705 (d) 735

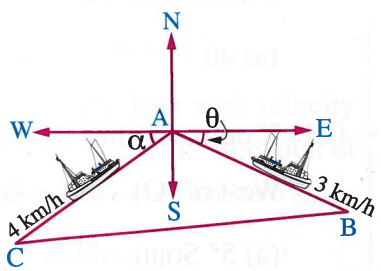
(25) A mobile network tower of height 30 m. is placed up on a building is observed from a point on the ground surface to find the two elevation angles of the top and base of the tower are of measures 60° and 30° respectively , then height of the building = ... m.

(a) 30 (b) 15 (c) 27.5 (d) 18

(26) From the base of a house of height 25 m. a person observed the top of an opposite minaret to find its elevation angle of measure 60° , then he ascended up on the top of the house and measured the elevation of the top of the same minaret to find it 40° , then the height of the minaret ≃ ..... m.

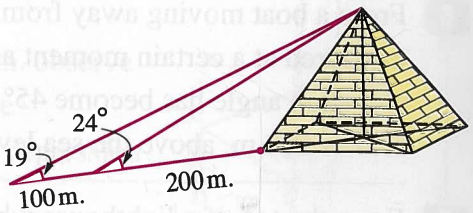
(a) 60 (b) 59 (c) $25 + 25\sqrt{3}$ (d) 50

(27) At exactly 7 a.m. a ship moved from a port in the direction θ° South of East with speed of 3 km./h. At exactly 8 a.m. another ship moved from the same port in the direction α° South of West with speed of 4 km./h. at exactly 11 a.m. the distance between the two ships was $12\sqrt{3}$ km. , then θ° + α° = ...



(a) 60° (b) 120° (c) 150° (d) 90°

(28) If a man measure the elevation angle of the top of the pyramid from two points on the ray that carries one of the pyramid's base diagonals and at a distance of 200 metres , 300 metres from one of the vertices of the base of the pyramid as shown in the figure.



He found that their measures are 24° and 19° respectivly , so the height of the pyramid to the nearest metre is equal to .....

(a) 124 (b) 152 (c) 168 (d) 212

Second Essay questions

1 From a point on the ground , a man observed the top of a tower at an angle of elevation of measure 25° When he walked horizontally towards the base of the tower x m. , the measure of the angle of elevation of the top became 55°
If the height of the tower is 85 m. , find the value of x to the nearest metre. « 123 m. »

2 From a point on the ground level , a man found that the measure of the elevation angle of the top of a rock is 75° The man walked on the horizontal road in the opposite direction of the base of the rock a distance 80 m. , he found that the measure of the elevation angle of the top of the rock became 52° If the two points of observation and the base of the rock are collinear, then find the height of the rock to the nearest metre. « 156 m. »

3 From the top of a rock of height 80 m. , the two angles of depression of the top and the base of a tower were measured to be 24° and 35° respectively.
Find the height of the tower to the nearest metre , knowing that the two bases of the rock and the tower are on the same horizontal level. « 29 m. »

4 From the top of a tower, a person measured the angles of depression of the top and the base of a minaret and their measures were 23° and 47° respectively.
Let the height of the minaret be 48 m., then find the distance between the two bases of the tower and the minaret to the nearest metre , knowing that the two bases are on one horizontal plane. « 74 m. »

5 From a window of a building 6 m. high , the elevation angle of the top of a tree was measured and found to be 15°, and the measure of the depression angle of the bottom of that tree was 30°. Find the height of the tree and its distance from the building.
« 8.8 m. , 10.4 m. »

6 From a boat moving away from a rock , the elevation angle of the top of the rock was measured at a certain moment and found to be 60° after two minutes , the measure of the elevation angle has become 45° Find the average velocity of the boat if the height of the rock is 500 m. above the sea level. « 105.7 m./min. »

7 From the top of a lighthouse whose height is 80 m. , a person measured the angles of depression of two boats in one horizontal plane , passing through the base of the lighthouse and they were of measure 50° 15̀ and 38° 40̀
Find the distance between the two boats if :
(1) The two boats are in two different sides of the lighthouse.
(2) The two boats are in the same side of the lighthouse. « 167 m. , 33 m. »

8 A lighthouse of height 60 m. is constructed on a hill close to the seashore. The two angles of elevation of the top and the base of the lighthouse were measured from a boat on the sea to be of measures 70° and 45° respectively.
Find the height of the hill from the sea level to the nearest metre. « 34 m. »

9 The measure of the angle of elevation of the top of a tower (under construction) from a point at a distance 130 metres from the base of the tower is 35° How many metres should the tower be risen such that the measure of the angle of elevation of its top from the same point would be 55° ? « 95 m. »

10 From a point on the ground surface , a man observed the angle of elevation of an airship moving vertically at a uniform velocity of a magnitude of 20 m./min. to find its measure is 35° and after three minutes , he observed the airship once again at the same point to find the measure of the angle of elevation of the airship is 15°. Find the distance between the man and the projection of the airship to the nearest metre. « 139 m. »

11 From the base of a house whose height is 20 m. , it was found that the angle of elevation of the top of a tower was of measure 25° Then it was found for a next time that the angle of elevation of the top of the same tower from the top of the house was of measure 18° Find the height of the tower. « 66 m. »

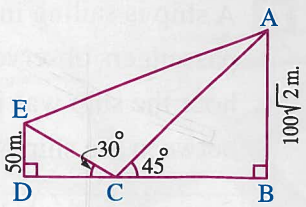
12 From the top of a mountain of height 100 m. , it was found that the measure of the depression angle of the top of a rock was 42° 37̀ Find the height of the rock if the distance between it and the mountain was 22 m. , knowing that they are on the same horizontal ground. « 79.8 m. »

13 The distance between two towers is 60 metres , the depression angle of the first tower top from the second tower top measures 30° If the height of the second tower is 150 metres. Find the height of the first tower. « 115 m. »

14 📖 **In the opposite figure :**

Two balloons A and E of heights $100\sqrt{2}$ m. and 50 m. observe an object (C) on the ground lying at the vertical plane passing through the two balloons. If the measures of depression angles of the object are 45° and 30° respectively.

, find the distance between the two balloons to the nearest metre.



« 246 m. »

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15 Two balloons at a height of 200 m. observed a body on the ground lying in the vertical plane which contains the two balloons. If the measures of the two depression angles of the body were 36° and 54° Find the distance between the two balloons knowing that the two balloons observe the body from opposite directions.

« 420.6 m. »

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16 From a point between two bases of a tower whose height is 75 m. and a rock whose height is 35 m. , it was found that the measures of the two angles of elevation of the top of the tower and the top of the rock were 63° and 48° respectively. Find :

(1) The distance between the two bases.

(2) The distance between the two tops.

« 70 m. , 80 m. »

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17 A ship moves with velocity 12 km./h. in the direction 40° South of the West , at the same time and place another ship moves with velocity 20 km./h. in the direction of the western North. Find the distance between the two ships after 3 hours from the beginning of their movement.

« 67 km. »

---

18 A man standing at a point B observed an object C in the east of B and at a distance 60 m. from B. After the man walked from B to A in the direction 60° North of the East, he found that the point C is in the direction 15° South of the East from A Find the distance CA

« 53.79 m. »

---

19 📖 From a point A on a riverbank , a man observed the position of a house at point B on the other riverbank to find it in the direction of 20° North of the East. As he walked parallel to the riverbank in the direction of East for a distance of 300 metres to reach point C , he found point B in the direction of 46° North of the East. Find the width of the river to the nearest metre known that the two riverbanks are parallel and the points A , B and C are at the same horizontal level.

« 168 m. »

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20 The port A lies on the North of the port B , where AB = 1000 m. If C is a ship lying in the direction 53° 43$\grave{}$ South of the East of A , and in the direction 42° 44$\grave{}$ North of the East of B , find the distance between the ship and the port B to the nearest metre.

« 596 m. »

21 A ship is sailing in direction of south with constant velocity of 24 km./h. One of the passengers observed that a fixed point lies in the direction 65° North of East. After one hour the ship was in the direction 79° South of West of the fixed point. Find the distance between the ship and the fixed point at this moment. « 42 km. »

22 A , B and C are three towns in the same horizontal plane, B lies to the western-south of A and at 40 km. away from it. If A lies in the direction 35° North of the East of C and B lies in the direction 5° North of the East of C. Find AC « 51 km. »

23 From a point on the horizontal plane passing through the base of a hill, a man found that the elevation angle of the top of the hill was of measure $27° 12\grave{}$ After he walked 2000 m. towards the hill on a plane inclined to the horizontal plane at an angle of measure 17°, he found that the measure of the elevation angle of the hill top became $36° 15\grave{}$. Calculate the height of the hill to the nearest metre. « 1916 m. »

24 A ship sails northeast with velocity of 24 k/h. A passenger in it observed two fixed points at 25° west of the North and after 4 hours, he found that one of the two points became at the direction of 23° South of the West while the other point became at the direction of 17° North of the West. Find the distance between the two points to the nearest kilometre given that the passenger and the points are at the same horizontal level. « 78 km. »

25 🕮 **Theoretical proof :** A pilot observed two observatory stations A and B on horizontal ground where AB = s metre to find the measures of their two angles of depression θ and α respectively. If the plane and the two observatory stations are at the same vertical level , the height of the plane on that time above the ground equals h metre and the vertical projection of the plane $\in \overline{AB}$

, prove that : $h = \frac{s}{\cot\theta + \cot\alpha}$

If $m(\angle\theta) = 48° 31\grave{}$, $m(\angle\alpha) = 75° 15\grave{}$, s = 1290 metres , find the value of h « 1124 m. »

26 🕮 **Theoretical proof :** A ladder of length L m. is placed by one of its ends on a vertical wall and the other end on a horizontal ground, the ladder inclined to the horizontal plane at an angle of measure α. The lower end moved far from the wall with a distance s , the measure of the inclined angle becomes θ Prove that : $L = \frac{s}{\cos\theta - \cos\alpha}$

If s = 40 cm. , $m(\angle\theta) = 30°$, $m(\angle\alpha) = 40°$, find the ladder length. « 4 m. »

Exercise 17

Trigonometric functions of the sum and difference between the measures of two angles

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\cos A \cos B + \sin A \sin B =$ .....

(a) $\sin (A + B)$ (b) $\sin (A - B)$ (c) $\cos (A + B)$ (d) $\cos (A - B)$

(2) $\sin A \cos B - \cos A \sin B =$ .....

(a) $\sin (A + B)$ (b) $\cos (A + B)$ (c) $\sin (A - B)$ (d) $\cos (A - B)$

(3) $\sin (A + B) + \sin (A - B) =$ .....

(a) $2 \sin A$ (b) $2 \sin^2 A$ (c) $2 \cos A \sin A$ (d) $2 \sin A \cos B$

(4) $\sin \frac{\pi}{9} \cos \frac{\pi}{18} + \cos \frac{\pi}{9} \sin \frac{\pi}{18} =$ .....

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{3}}{2}$

(5) If $\frac{\tan X - \tan Y}{1 + \tan X \tan Y} = 2$, then $\tan (X - Y) =$ .....

(a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{3}$

(6) $\frac{\tan 50° + \tan 10°}{1 - \tan 50° \tan 10°} =$ .....

(a) $\tan 40°$ (b) $\tan 60°$ (c) $\tan 50° \times \tan 10°$ (d) $2 \tan 50° \tan 10°$

(7) $\frac{1 + \tan A}{1 - \tan A} =$ .....

(a) $\tan \left(\frac{\pi}{4} + A\right)$ (b) $\tan \left(\frac{3\pi}{4} + A\right)$ (c) $\tan \left(\frac{\pi}{2} + A\right)$ (d) $\tan \left(\frac{5\pi}{4} - A\right)$

(8) If $\tan 20° = a$, $\cot 15° = b$, then $\tan 35°$ = ...

(a) $\frac{a\,b-1}{b-a}$ (b) $\frac{a-b}{a\,b-1}$ (c) $\frac{a\,b+1}{b-a}$ (d) $\frac{a\,b+1}{b+a}$

(9) sin $\left(\theta+\frac{\pi}{6}\right)$ = ...

(a) $\frac{1}{2}\left(\cos\theta+\sqrt{3}\sin\theta\right)$ (b) $\frac{1}{2}(\cos\theta+\sin\theta)$

(c) $\frac{1}{2}\left(\sqrt{3}\cos\theta+\sin\theta\right)$ (d) $\frac{1}{2}\left(\sqrt{3}\cos\theta+\sqrt{2}\sin\theta\right)$

(10) $\sin 5X \sin 3X + \cos 5X \cos 3X$ = ...

(a) $\cos 2X$ (b) $\cos 8X$ (c) $\sin 8X$ (d) $\sin 2X$

(11) If $\tan\theta = \frac{1}{2}$, then $\tan(\theta+45°)$ = ...

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 3 (d) 6

(12) An acute-angled triangle ABC , where $\sin A = \frac{3}{10\cos B}$, $\cos A = \frac{1}{5\sin B}$, then m ($\angle$ C) = ...

(a) 15° (b) 45° (c) 30° (d) 60°

(13) Δ ABC in which : $\tan\frac{A}{2} = \frac{1}{3}$, $\tan\frac{B}{2} = \frac{2}{3}$, then $\tan\frac{C}{2}$ = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{7}{9}$ (d) 1

(14) If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\tan(A+B)$ = ...

(a) 1 (b) $-\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

(15) cot $\left(2\pi-\frac{5\pi}{12}\right)$ = ...

(a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$ (c) $-2-\sqrt{3}$ (d) $\sqrt{3}-2$

(16) $\sin 87° \cos 27° - \cos 87° \sin 27°$ = ...

(a) $\sin 60°$ (b) $\cos 60°$ (c) $\sin 114°$ (d) $\cos 114°$

(17) S.S. of the equation : $\sin X \cos 45° - \cos X \sin 45° = \frac{1}{2}$ where $0° < X < 360°$ equals .....

(a) $\{30°, 150°\}$ (b) $\{30°\}$ (c) $\{75°, 195°\}$ (d) $\{75°, 105°\}$

(18) $\sqrt{2}\cos(A-45°)$ = ...

(a) $\sin A + \cos A$ (b) $\sin A - \cos A$ (c) $\sin A \cos A$ (d) $\tan A + \cot A$

(19) If $\angle$ A supplements $\angle$ B , then $\cos A \cos B - \sin A \sin B$ = ...

(a) -1 (b) 180° (c) 1 (d) zero

(20) If $\cos A = 0.6$, $\cos B = 0.8$ where A and B are measures of acute angles , then $\cos (A - B) =$ .....

(a) $\frac{24}{25}$ (b) $\frac{7}{24}$ (c) $\frac{7}{25}$ (d) $\frac{2\sqrt{6}}{5}$

(21) $\cos X \cos\left(X + \frac{\pi}{3}\right) + \sin X \sin\left(X + \frac{\pi}{3}\right) =$ .....

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{-1}{2}$

(22) If $\tan A + \tan B = 4 - 4 \tan A \tan B$, then $\tan (A + B) =$ .....

(a) $\frac{1}{2}$ (b) 4 (c) 1 (d) 2

(23) $\frac{\tan (A - B) + \tan B}{1 - \tan (A - B) \tan B} =$ .....

(a) $\tan A$ (b) $-\tan (A + B)$ (c) $\tan (A + B)$ (d) $-\tan B$

(24) In ΔABC , $\cos A = \frac{-3}{5}$, $\sin B = \frac{5}{13}$, then $\sin C =$ .....

(a) $\frac{32}{65}$ (b) $\frac{17}{65}$ (c) $\frac{33}{65}$ (d) $\frac{3}{13}$

(25) If $\angle A$, $\angle B$ are acute angles , $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$, then $A + B =$ .....

(a) 30° (b) 45° (c) 60° (d) 90°

(26) If $\tan x + \tan y = 4$, $\cot x + \cot y = \frac{2}{3}$, then $\tan (x + y) =$ .....

(a) $\frac{-3}{4}$ (b) $\frac{-4}{5}$ (c) $\frac{3}{5}$ (d) 1

(27) $\sin (X - 60°) + \cos (X - 30°) =$ .....

(a) $\sin X$ (b) $\cos X$ (c) $\sin (X + 30°)$ (d) $\cos (X + 60°)$

(28) If $\cos (A - B) = \frac{3}{5}$, $\tan A \tan B = 2$, then .....

(a) $\cos A \cos B = \frac{1}{5}$ (b) $\sin A \sin B = \frac{-2}{5}$

(c) $\cos (A + B) = \frac{1}{5}$ (d) $\sin A \sin B = \frac{4}{5}$

(29) If $x + y = \frac{5\pi}{6}$, then $(\sin x - \cos y)^2 + (\cos x - \sin y)^2 =$ .....

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

(30) ABC is a triangle in which : $\sin A + \sin B \cos C + \cos B \sin C = \sqrt{3}$, then m ($\angle A$) could be equal .....

(a) 30° (b) 45° (c) 60° (d) 90°

(31) $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} =$ .....

(a) $\sec x$ (b) $\csc x$ (c) $\sin^2 x$ (d) $\cos^2 x$

(32) $\tan A - \tan B = \frac{\dots\dots\dots}{\cos A \cos B}$

(a) $\sin (A + B)$ (b) $\sin (A - B)$ (c) $\cos (A - B)$ (d) $\sin A - \sin B$

(33) If $\sec A = \frac{5}{4}$, $\csc B = \frac{13}{5}$, where A and B are the measures of two acute angles , then $\sec (A - B) =$ .....

(a) $\frac{56}{65}$ (b) $\frac{63}{65}$ (c) $\frac{-33}{65}$ (d) $\frac{65}{63}$

(34) If $\tan^2 A = \frac{9}{16}$, where $A \in [0 , \frac{\pi}{2}[$, $\tan^2 B = \frac{1}{9}$ where $B \in [\frac{\pi}{2} , \pi[$, then $\tan (A + B) =$ .....

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

(35) If $\tan A = x + 1$, $\tan B = x - 1$, then $\cot (A - B) =$ .....

(a) $\frac{2}{x^2}$ (b) $\frac{1}{2} x^2$ (c) x^2 (d) $\frac{1}{x^2}$

(36) If A and B are the measures of two acute angles and $\sin A \sin B = \frac{1}{2}$, $\cos A \cos B = \frac{1}{3}$, then $\cos (A - B) =$ .....

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{-1}{6}$

(37) If $\tan (\theta + 45°) = \frac{3}{2}$, then $\tan \theta =$ .....

(a) $-\frac{1}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) 1

(38) If $0 < y < \frac{\pi}{2}$, $x + y = \frac{\pi}{4}$, $\sin y = \frac{1}{\sqrt{10}}$, then $\tan (3x + 4y) =$ .....

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) -1

(39) If $\tan (C - 30°) = \frac{a \tan C - 1}{a + \tan C}$, then $a =$ .....

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $-\sqrt{3}$

(40) If $x \in [0 , 2\pi[$, and $\frac{\tan x - \cot 55°}{1 + \tan x \cot 55°} = 1$, then $x =$ .....

(a) $\frac{\pi}{9}$ (b) $\frac{4\pi}{9}$ (c) $\frac{4\pi}{9}$ or $\frac{13\pi}{9}$ (d) $\frac{10\pi}{9}$ or $\frac{11\pi}{9}$

(41) If $1 + \cos x \cos y + \sin x \sin y = 0$, then .....

(a) $x - y = -1$ (b) $x + y = 1$

(c) $\cos x - \cos y = 0$ (d) $\cos x + \cos y = 0$

(42) If $\cot x - \cot y = -5$, $\tan x \tan y = \frac{1}{2}$, then $\tan (x - y) =$ .....

(a) $\frac{1}{3}$ (b) $\frac{5}{3}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

(43) If $x + y = 225°$, then $(1 + \tan x)(1 + \tan y) =$ .....

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{2}$

(44) If $0 \leq x < \pi$, then solution set of the equation : $\sin(x - \frac{4\pi}{3}) = 2 \sin(x - \frac{\pi}{3})$ is .....

(a) $\{\frac{\pi}{2}\}$ (b) $\{\frac{\pi}{3}\}$ (c) $\{\frac{\pi}{4}\}$ (d) $\{\frac{\pi}{6}\}$

(45) If $A + B = 3(C + D) = 180°$

, then $\frac{\sin A \sin B + \sin(270° - A) \sin(90° - B)}{\cos C \cos D - \cos(270° + C) \sin(180° - D)} =$ .....

(a) 1 (b) 2 (c) -1 (d) $\frac{\sqrt{3}}{2}$

(46) In ΔABC : $\csc A (\sin B \cos C + \cos B \sin C) =$ .....

(a) $\frac{c}{a}$ (b) $\frac{a}{c}$ (c) 1 (d) $\tan A$

(47) $\frac{\sin(A - B)}{\sin A \sin B} + \frac{\sin(B - C)}{\sin B \sin C} + \frac{\sin(C - A)}{\sin C \sin A} =$ .....

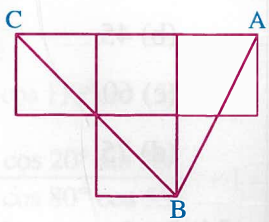
(a) zero (b) 1

(c) $\sin A + \sin B + \sin C$ (d) $\sin A \sin B \sin C$

(48) In the opposite figure :

Four congruent squares ,

then $\tan(\angle ABC) =$ .....



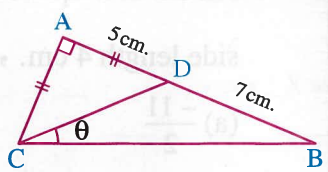
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

(49) In the opposite figure :

If ABC is a right-angled triangle at A , BD = 7 cm.

, AD = AC = 5 cm. , then $\tan\theta =$ .....



(a) $\frac{7}{17}$ (b) $\frac{5}{13}$

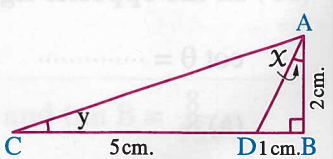
(c) $\frac{5}{12}$ (d) $\frac{12}{35}$

(50) In the opposite figure :

ABC is right-angled triangle at B

where AB = 2 cm. , BD = 1 cm. , CD = 5 cm.

, then $x + y =$ .....

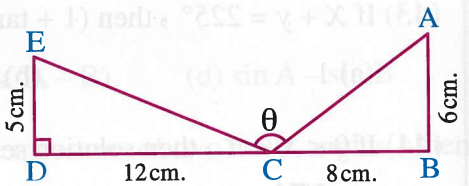


(a) 15° (b) 30°

(c) 45° (d) 60°

(51) In the opposite figure :

If $C \in \overline{BD}$, then $\sin\theta$ = ...



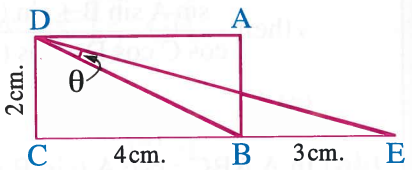
(a) $\frac{56}{65}$ (b) $\frac{16}{65}$

(c) $\frac{64}{65}$ (d) $\frac{2}{7}$

(52) In the opposite figure :

ABCD is a rectangle , $E \in \overrightarrow{CB}$

, where BE = 3 cm. , then $\tan\theta$ = ...

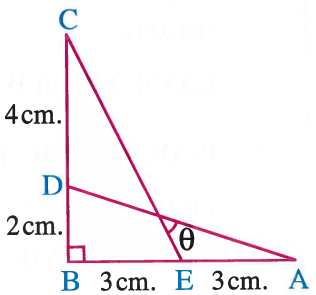


(a) $\frac{3}{16}$ (b) $\frac{1}{2}$

(c) $\frac{2}{7}$ (d) $\frac{3}{14}$

(53) In the opposite figure :

θ = ... °



(a) 30

(b) 45

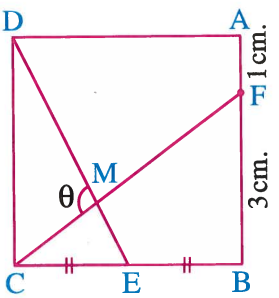
(c) 60

(d) 75

(54) In the opposite figure :

ABCD is a square whose

side length 4 cm. , then $\tan\theta$ = ...

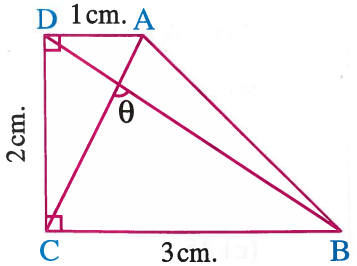


(a) $\frac{-11}{2}$ (b) 3

(c) $\frac{-3}{8}$ (d) $\frac{23}{3}$

(55) In the opposite figure :

$\cot\theta$ = ...



(a) 8 (b) 4

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Second Essay questions

1 **Without using the calculator , find each of the following :**

(1) cos 105° « $\frac{\sqrt{2}-\sqrt{6}}{4}$ »

(2) sin 75° « $\frac{\sqrt{2}+\sqrt{6}}{4}$ »

(3) cos 345° « $\frac{\sqrt{6}+\sqrt{2}}{4}$ »

(4) tan (– 75°) « $-2-\sqrt{3}$ »

(5) sin $\frac{11\pi}{12}$ « $\frac{\sqrt{6}-\sqrt{2}}{4}$ »

(6) tan $\frac{7\pi}{12}$ « $-2-\sqrt{3}$ »

2 **Without using the calculator , find each of the following in its simplest form :**

(1) cos 105° cos 15° + sin 105° sin 15° « Zero »

(2) sin 75° cos 15° + cos 75° sin 15° « 1 »

(3) sin $\frac{7\pi}{12}$ cos $\frac{5\pi}{12}$ – cos $\frac{7\pi}{12}$ sin $\frac{5\pi}{12}$ « $\frac{1}{2}$ »

(4) cos $\frac{5\pi}{12}$ cos $\frac{\pi}{12}$ – sin $\frac{5\pi}{12}$ sin $\frac{\pi}{12}$ « Zero »

(5) cos 13° 58̂ cos 31° 2̂ – sin 13° 58̂ sin 31° 2̂ « $\frac{1}{\sqrt{2}}$ »

(6) $\frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°}$ « 1 »

(7) $\frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{12}}$ « –1 »

(8) $\frac{\tan 15° - \tan 150°}{1 + \tan 15° \tan 150°}$ « 1 »

(9) sin 110° sin 130° – cos 110°cos 130° « $\frac{1}{2}$ »

(10) sin 39° cos 9° – sin 51° cos 81° « $\frac{1}{2}$ »

(11) $\frac{\sin 20° \cos 25° + \cos 20° \sin 25°}{\cos 10° \cos 35° - \cos 80° \cos 55°}$ « 1 »

(12) $\frac{\tan 25° + \cot 70°}{1 - \cot 65° \tan 20°}$ « 1 »

3 **Simplify each of the following :**

(1) cos 3 X cos 2 X – sin 3 X sin 2 X « cos 5 X »

(2) sin (X + Y) cos Y – cos (X + Y) sin Y « sin X »

(3) sin (25° + X) cos (20° – X) + cos (25° + X) sin (20° – X) « $\frac{1}{\sqrt{2}}$ »

(4) cos (A – B) – cos (A + B) « 2 sin A sin B »

(5) $\frac{\tan (40° - X) + \tan (5° + X)}{1 - \tan (40° - X) \tan (5° + X)}$ « 1 »

4 **If A and B are measures of acute angles where cos A = $\frac{3}{5}$ and tan B = $\frac{8}{15}$, find without using the calculator each of the following :**

(1) sin (A + B)

(2) tan (A – B)

(3) sec (A – B)

(4) cot (A + B) « $\frac{84}{85}$, $\frac{36}{77}$, $\frac{85}{77}$, $\frac{13}{84}$ »

5 If $5\cos X + 3 = 0$ where $X \in]0, \pi[$ and $\cos Y = \frac{12}{13}$ where $Y \in]\pi, 2\pi[$, then find the value of each of the following :

(1) $\sin(X - Y)$ (2) $\cos(X - Y)$ (3) $\tan(X + Y)$ « $\frac{33}{65}$, $\frac{-56}{65}$, $\frac{-63}{16}$ »

6 If $\sin A = \frac{3}{5}$ where $0° < A < 90°$ and $\tan B = -7$ where $90° < B < 180°$, prove that : $A + B = 135°$

7 If $\sin A = \frac{-8}{17}$ where $180° < A < 270°$ and $\cos B = \frac{-5}{13}$ where $90° < B < 180°$, find without using the calculator each of the following :

(1) $\cos(A - B)$ (2) $\sin(A + B)$ (3) $\tan(A - B)$

« $\frac{-21}{221}$, $\frac{-140}{221}$, $\frac{-220}{21}$ »

8 If $\sec A = \frac{13}{5}$ where $0 < A < \frac{\pi}{2}$ and $\tan B = \frac{-3}{4}$ where $B \in]\frac{\pi}{2}, \pi[$, then find :

(1) $\sin(A - B)$ (2) $\cos(A + B)$ (3) $\cot(A - B)$

« $\frac{-63}{65}$, $\frac{-56}{65}$, $\frac{-16}{63}$ »

9 Creative thinking : If $\tan A = \frac{2}{3}$ where $\pi < A < \frac{3\pi}{2}$ and $\tan B = \frac{1}{5}$ where $0 < B < \frac{\pi}{2}$, then find the value of each of : $\tan(A + B)$ and $\cos(A + B)$, then from that prove that : $A + B = \frac{5\pi}{4}$ « 1 , $\frac{-\sqrt{2}}{2}$ »

10 ABC is a triangle in which , $\tan A = \frac{1}{7}$ and $\tan B = \frac{3}{4}$ Without using calculator , find : $m(\angle C)$ « 135°. »

11 If $m(\angle X) + m(\angle Y) + m(\angle Z) = \frac{\pi}{2}$, prove that : $\tan X \tan Y + \tan Y \tan Z + \tan Z \tan X = 1$

12 If $\tan(A + B) = 33$ and $\tan A = 3$, prove that : $\tan B = 0.3$

13 In Δ ABC , $\tan A = \frac{n}{n+1}$ and $\tan B = \frac{1}{2n+1}$ where $n \in \mathbb{R}^+$ Prove that : $\tan(A + B) = 1$

14 If $\frac{\cos(A+B)}{\cos(A-B)} = \frac{1}{3}$, prove that : $2\sin A \sin B = \cos A \cos B$, then prove that : $2\tan A = \cot B$ and if $\tan A = \frac{2}{5}$, find : $\tan B$ and $\tan(A - B)$ « $\frac{5}{4}$, $\frac{-17}{30}$ »

15 **Prove that :**

(1) $\tan 75° = 1 + \tan 30° + \tan 30° \tan 75°$

(2) $\sqrt{2}\cos\left(\frac{\pi}{4} - X\right) = \cos X + \sin X$

(3) $\cos\left(X - \frac{\pi}{6}\right)\cos X + \cos\left(\frac{2\pi}{3} - X\right)\cos\left(X - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$

(4) $\cos(120° - A) = \frac{1}{2}\left(\sqrt{3}\sin A - \cos A\right)$

(5) $\tan(45° - A) = \frac{\cos A - \sin A}{\cos A + \sin A}$

(6) $\frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \tan A$

(7) $\frac{\tan A + \tan B}{\tan(A+B)} + \frac{\tan A - \tan B}{\tan(A-B)} = 2$

(8) $\tan(A+B) = \frac{\cot A + \cot B}{\cot A \cot B - 1}$

(9) $\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}$

(10) $\tan(A+B)\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$

(11) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

16 **Without using the calculator , prove that :**

(1) $\sin 35° + \cos 65° = \cos 5°$

(2) $\cos 52° + \cos 68° = \sin 82°$

(3) $\sin 67° + \cos 37° = \sqrt{3}\cos 7°$

(4) $\tan 40° = \frac{1 - \tan 5°}{1 + \tan 5°}$

17 **If $X \in [0 , 2\pi[$, find the value of X in each of the following :**

(1) $\sin X \cos 15° + \cos X \sin 15° = \frac{1}{2}$ « 15° or 135° »

(2) $\sin X \cos 20° - \cos X \sin 20° = \frac{1}{2}$ « 50° or 170° »

(3) $\cos 2X \cos X + \sin 2X \sin X = \frac{\sqrt{3}}{2}$ « 30° or 330° »

(4) $\frac{\tan X - \tan 25°}{1 + \tan X \tan 25°} = 1$ « 70° or 250° »

(5) $\tan X + \tan 20° + \tan X \tan 20° = 1$ « 25° or 205° »

(6) $\sin(60° + X) = 2\sin X$ « 30° or 210° »

(7) $\sin(X + 30°) = 2\cos X$ « 60° or 240° »

(8) $\sin\left(X + \frac{\pi}{4}\right) + \sin\left(X - \frac{\pi}{4}\right) = -1$ « 225° or 315° »

18 **If $0 < \theta < \frac{\pi}{2}$, find the solution set of each of the following :**

(1) $\sin 135° \cos\theta - \cos 135° \sin\theta = 1$ « {45°} »

(2) $\frac{\tan 4\theta - \tan 3\theta}{1 + \tan 4\theta \tan 3\theta} = \sqrt{3}$ « {60°} »

(3) $\sqrt{3}\cos\theta - \sin\theta = 1$ « {30°} »

(4) $\sin(45° + 2\theta)\sin(60° + \theta) + \sin(45° - 2\theta)\sin(30° - \theta) = 1$ « {15°} »

19 **In ΔABC, prove that :**

(1) $\sin\left(\frac{A+B}{2}\right)\cos\frac{C}{2} + \cos\left(\frac{A+B}{2}\right)\sin\frac{C}{2} = 1$

(2) $\cos\left(\frac{A+B-C}{2}\right)\cos C + \sin\left(\frac{C-A-B}{2}\right)\sin C = 0$

(3) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(4) $\tan\frac{3A}{2}\tan\frac{3B}{2} + \tan\frac{3B}{2}\tan\frac{3C}{2} + \tan\frac{3C}{2}\tan\frac{3A}{2} = 1$

20 ABC is a right-angled triangle at B, in which AB = 4 cm., BC = 3 cm. and $\overline{CD}$ is a median. **Find :** $\cos(\angle ACD)$ « $\frac{17\sqrt{13}}{65}$ »

21 **Electricity :** If the electric current intensity is given by the relation $C = \frac{3}{2}\cos 285° t$

(1) Rewrite the previous relation using the difference between the measures of two angels.

(2) Find the electric current intensity after one second without using the calculator.

Third Problems that measure high standard levels of thinking

1 **Choose the correct answer from those given :**

(1) If $f(x) = \tan x$, then $f\left(x + \frac{\pi}{4}\right) \times f\left(x - \frac{\pi}{4}\right) =$ .....

(a) – 1 (b) 1 (c) 2 tan x (d) cot x

(2) In triangle ABC, $\tan A = 3 + x$, $\tan B = 3 - x$, $\tan C = 3$, then $x =$ .....

(a) ± 1 (b) $\pm\sqrt{6}$ (c) ± 6 (d) $\pm\sqrt{10}$

(3) If $\sin x - \cos y = \frac{1}{6}$, $\cos x + \sin y = \frac{5}{6}$, then $\sin(x - y) =$ .....

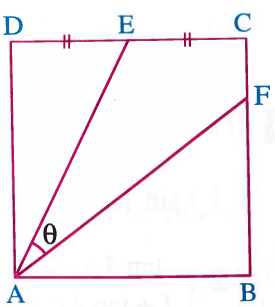
(a) $\frac{19}{24}$ (b) $\frac{17}{23}$ (c) $\frac{23}{36}$ (d) $\frac{25}{48}$

(4) **In the opposite figure :**

ABCD is a square, E is the midpoint of $\overline{CD}$, BC = 4 CF, then $\cos\theta =$ .....

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$

(c) $\frac{3}{\sqrt{5}}$ (d) $\frac{4}{\sqrt{5}}$



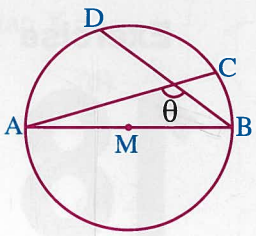
(5) In the opposite figure :

$\overline{AB}$ is a diameter of circle M , its length = 25 cm.

$\overline{AC}$ is a chord , its length = 24 cm.

$\overline{BD}$ is a chord , its length = 20 cm.

, then $\tan\theta$ = ...



(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

(6) Δ ABC which is right angled at A

, then $\frac{b\cos B + c\cos C}{\cos(B - C)}$ = ...

(a) a + b (b) a + c (c) a (d) c

(7) If α and β are measures of acute angles and $\cos(\alpha + \beta) = \frac{12}{13}$, then $\sin(\alpha - \beta) = \frac{3}{5}$

, then $\sin 2\alpha$ = ...

(a) $\frac{16}{15}$ (b) zero (c) $\frac{56}{65}$ (d) $\frac{64}{65}$

---

2 If A , B , C are the measures of three acute angles , $\sin A = \frac{1}{\sqrt{5}}$, $\cos B = \frac{4}{5}$ and $\tan C = \frac{1}{2}$

, **prove that :** $A + B + C = 90°$ [without using the calculator]

---

3 If $\tan A$ and $\tan B$ are the two roots of $2X^2 + 3X - 1 = 0$, so without using the calculator

, **find :** $\tan(A + B)$ and hence find $[m(\angle A) + m(\angle B)]$ « – 1 , 135° or 315° »

---

4 **If $X \in [0 , 2\pi[$, find the value of X that makes the expression**

$\sin X \cos 55° + \cos X \sin 55°$

(1) Is the greatest. **(2)** Is the smallest. « 35° , 215° »

---

5 ABC is an acute-angled triangle , $\tan A = 0.75$, $\tan B = 2.4$

, **prove that :** a : b : c = 13 : 20 : 21

---

6 If $A \in]0 , \frac{\pi}{2}[$ and $\sin A = \frac{\sqrt{3}+1}{2\sqrt{2}}$, then find without using the calculator : $m(\angle A)$

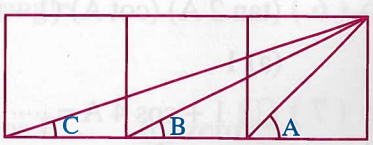
« 75° »

---

7 **In the opposite figure :**

Three squares.

Prove that : $m(\angle A) = m(\angle B) + m(\angle C)$



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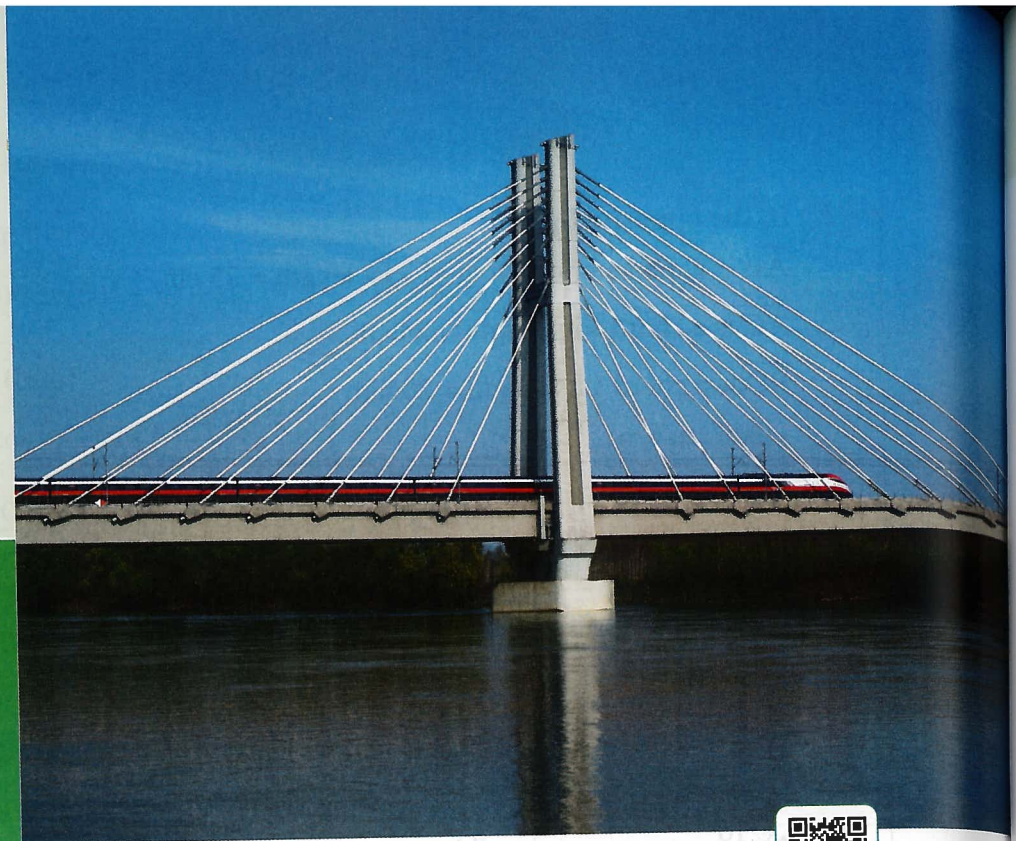
8 If $\sin(A + B) = \frac{3}{5}$ and $\sin(A - B) = \frac{1}{10}$

, **prove that :** $5\tan A = 7\tan B$

Exercise 18

Trigonometric functions of double of the measure of an angle



From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) $\cos^2 X - \sin^2 X =$ .....

(a) 1 (b) $\sin 2X$ (c) $\cos 2X$ (d) $2 \sin X \cos X$

(2) $\sin 2A =$ .....

(a) $2 \sin A$ (b) $2 \sin A \cos A$ (c) $1 - 2 \sin^2 A$ (d) $\sin^2 A$

(3) $\tan 2A =$ .....

(a) $2 \tan A$ (b) $2 \tan A \cot A$ (c) $\frac{2 \tan A}{1 - \tan^2 A}$ (d) $\frac{2 \tan A}{1 + \tan^2 A}$

(4) $\sin 6X \cos 6X =$ .....

(a) $\sin 12X$ (b) $\frac{1}{2} \sin 12X$ (c) zero (d) $\sin 3X$

(5) If $\frac{\tan X}{1 - \tan^2 X} = \frac{5}{2}$, then $\tan 2X =$ .....

(a) $\frac{5}{2}$ (b) 5 (c) $\frac{2}{5}$ (d) 2

(6) $(\tan 2A)(\cot A)(1 - \tan^2 A) =$ .....

(a) 1 (b) $\sin A$ (c) $\tan A$ (d) 2

(7) 1 + $\cos 4A =$ .....

(a) $2 \cos^2 4A$ (b) $\cos^2 2A$ (c) $\cos^2 4A$ (d) $2 \cos^2 2A$

(8) $1 - 2 \sin^2 50° =$ .....

(a) $\sin 100°$ (b) $\cos 50°$ (c) $\cos 100°$ (d) $\sin 50°$

(9) If $\tan 2X = \frac{4}{3}$ where X is the measure of an acute angle , then $\tan X =$ .....

(a) -2 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(10) If $\cos C = \frac{1}{3}$, then $\cos 2C =$ .....

(a) zero (b) $-\frac{2}{3}$ (c) $-\frac{7}{9}$ (d) $\frac{2}{3}$

(11) If $\sin 2\mathcal{X} = \frac{5}{13}$, then $\tan \mathcal{X} + \cot \mathcal{X} =$ .....

(a) $\frac{19}{5}$ (b) 4 (c) $\frac{24}{5}$ (d) $\frac{26}{5}$

(12) $\cos^2 C - \cos 2C =$ .....

(a) $\sin C$ (b) $\cos C$ (c) $\sin^2 C$ (d) $\tan C$

(13) $2\cos^2\left(\frac{\pi}{4} - \frac{3\mathcal{X}}{2}\right) - 1 =$ .....

(a) $\cos 3\mathcal{X}$ (b) $\sin 3\mathcal{X}$ (c) $\cos \frac{3}{2}\mathcal{X}$ (d) $\sin \frac{3}{2}\mathcal{X}$

(14) $2\sin(\mathcal{X} + 45°)\cos(\mathcal{X} + 45°) =$ .....

(a) $2\sin(\mathcal{X} + 45°)$ (b) $\cos(90° + 2\mathcal{X})$ (c) $\sin 2\mathcal{X}$ (d) $\cos 2\mathcal{X}$

(15) $(1 + \cos 2\mathcal{X})\tan \mathcal{X} =$ .....

(a) $\sec 2\mathcal{X}$ (b) $\cot 2\mathcal{X}$ (c) $\sin 2\mathcal{X}$ (d) $\cos 2\mathcal{X}$

(16) $\left(\sin \frac{\mathcal{X}}{2} - \cos \frac{\mathcal{X}}{2}\right)^2 =$ .....

(a) $1 - \cos \mathcal{X}$ (b) $1 - \cos \frac{\mathcal{X}}{2}$ (c) $1 - \sin \mathcal{X}$ (d) $1 - \sin \frac{\mathcal{X}}{2}$

(17) If $\sin \mathcal{X} + \cos \mathcal{X} = 1$, then $\sin 2\mathcal{X} =$ .....

(a) 3 (b) 2 (c) 1 (d) zero

(18) $\csc 2\mathcal{X} =$ .....

(a) $\sec \mathcal{X} \csc \mathcal{X}$ (b) $\frac{1}{2}\sec \mathcal{X} \csc \mathcal{X}$ (c) $\sin \mathcal{X} \cos \mathcal{X}$ (d) $2\sin \mathcal{X} \cos \mathcal{X}$

(19) $(\sin \mathcal{X} - \cos \mathcal{X})(\sin \mathcal{X} + \cos \mathcal{X}) = \frac{3}{5}$, then $\cos 2\mathcal{X} =$ .....

(a) 1 (b) $\frac{3}{5}$ (c) $\frac{-3}{5}$ (d) $\frac{9}{25}$

(20) If $A + B = 180°$ where B is the measure of a positive acute angle , then $\sin(A - B) =$ .....

(a) $\sin A \cos B$ (b) $\tan B \cot A$ (c) $2\sin B \cos B$ (d) zero

(21) $\csc^2 \mathcal{X}(1 - \cos 2\mathcal{X}) =$ .....

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) -2

(22) $\frac{\tan\theta + \cot\theta}{\csc 2\theta}$ = ...

(a) zero (b) 1 (c) 2 (d) $\cot 2\theta$

(23) $\frac{\sin 4\theta}{1 + \cos 4\theta}$ = ...

(a) $\sin 2\theta$ (b) $\cos 2\theta$ (c) $\tan 2\theta$ (d) $\cot 2\theta$

(24) If $\cos^4 x - \sin^4 x = \cos a x$, then a = ...

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

(25) $\frac{1}{2}(\tan\theta + \cot\theta)$ = ...

(a) $\cos 2\theta$ (b) $\sin 2\theta$ (c) $\csc 2\theta$ (d) $\sec 2\theta$

(26) If $\tan x - \cot x = 3$, then $\tan 2x$ = ...

(a) 6 (b) $\frac{2}{3}$ (c) $\frac{-2}{3}$ (d) $\frac{3}{2}$

(27) If $\frac{\sin x}{1 + \cos x} = a$, then $\cot\frac{x}{2}$ = ...

(a) $\frac{1}{2}$ a (b) a (c) $\frac{1}{a}$ (d) $\frac{2}{3}$ a

(28) If $x \in]0, \frac{\pi}{2}[$ and $\sin x = \frac{1}{\sqrt{5}}$, then $\sin^2\left(\frac{\pi}{4} - x\right) - \cos^2\left(\frac{\pi}{4} - x\right)$ = ...

(a) $\frac{3}{5}$ (b) $\frac{-4}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{4}{5}$

(29) If $\cos B = \frac{-4}{5}$ where $\pi < B < \frac{3\pi}{2}$, then $\frac{\sin 2B}{2\cos 2B}$ = ...

(a) $\frac{7}{12}$ (b) $\frac{3}{4}$ (c) $\frac{24}{7}$ (d) $\frac{12}{7}$

(30) If $x \in]0, \frac{\pi}{2}[$ and $\tan x = \frac{1}{3}$, then $\frac{1 - \cos 2x}{1 + \cos 2x}$ = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

(31) $\frac{1 - \tan^2 x}{1 + \tan^2 x}$ = ...

(a) $\cos 2x$ (b) $\sin 2x$ (c) $\cos x$ (d) $\sin x$

(32) $\frac{1 + \sin 2x}{\sin x + \cos x}$ = ...

(a) $\sin x$ (b) $\cos x$ (c) $\sin x + \cos x$ (d) $\tan x$

(33) If $2\sin x \cos^3 x - 2\cos x \sin^3 x = \frac{1}{2}$, then $\sin 4x$ = ...

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(34) If $\sin A \cos^3 A + \cos A \sin^3 A = \frac{1}{2}$, then A = ...

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) π

(35) If $\sin 32° = x$, then $\sin 4° \cos 4° \cos 8° \cos 16° =$ .....

(a) $\frac{x}{16}$ (b) $\frac{x}{8}$ (c) $\frac{x}{4}$ (d) $\frac{x}{2}$

(36) If $\tan A = \frac{1}{2}$, then $\cot \frac{A}{2} - \tan \frac{A}{2} =$ .....

(a) 2 (b) 4 (c) 3 (d) 5

(37) If $\tan \theta = \frac{-5}{12}$, $\theta \in]0 , \pi[$, then $\cos \frac{\theta}{2} =$ .....

(a) $\frac{12}{13}$ (b) $\frac{-12}{13}$ (c) $\frac{1}{\sqrt{26}}$ (d) $\frac{5}{\sqrt{26}}$

(38) If $\sin x \cos x = \frac{\sqrt{3}}{4}$, $x \in]45° , 90°[$, then $\tan 4x =$ .....

(a) $\frac{\sqrt{3}}{2}$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $\sqrt{6}$

(39) If $4 \cos 2C + 3 \sin 2C = 0$ where C is measure of an acute positive angle , then $\tan C =$ .....

(a) $\frac{-4}{3}$ (b) $\frac{-2}{\sqrt{3}}$ (c) 1 (d) 2

(40) If $B - 2A = 180°$, $\tan A = \frac{1}{2}$, then $\tan B =$ .....

(a) $\frac{4}{3}$ (b) $\frac{1}{4}$ (c) $\frac{2}{\sqrt{3}}$ (d) 1

(41) If A , B , C are measures of angles of ΔABC , $\tan C = \frac{3}{4}$, then $\sin\left(\frac{A+B}{2}\right) =$ .....

(a) 0.3 (b) $\frac{3}{\sqrt{10}}$ (c) $\frac{3}{8}$ (d) 0.9

(42) If $f(x) = \tan x$, then $f\left(x + \frac{\pi}{4}\right) + f\left(x - \frac{\pi}{4}\right) =$ .....

(a) $\tan 2x$ (b) $4 \tan x$ (c) $2 \tan 2x$ (d) $2 \cot 2x$

(43) $\cot \theta - \cot 2\theta =$ .....

(a) $\sin \theta$ (b) $\sin 2\theta$ (c) $\csc \theta$ (d) $\csc 2\theta$

(44) $\frac{\cos 2x}{\cos x - \sin x} - \sin x =$ .....

(a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\sec x$

(45) $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} =$ .....

(a) 1 (b) 2 (c) $\sin x$ (d) $\cos x$

(46) $\frac{2 \sin x - \sin 2x}{\sin^3 x} =$ .....

(a) $\cos^2 \frac{x}{4}$ (b) $\tan^2 \frac{x}{2}$ (c) $\sin^2 \frac{x}{2}$ (d) $\sec^2 \frac{x}{2}$

(47) If $\chi \in \left[0 , \frac{\pi}{4}\right]$ and $\left(16^{\cos \chi}\right)^{\sin \chi} = 2$, then $\tan 2\chi =$ .....

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{3}$ (c) $\frac{1}{2}$ (d) 2

(48) If $1 + \cot^2 \theta = \frac{m}{\sin^2 2\theta}$, then m = ...

(a) $\cos^2 \theta$ (b) $4 \cos^2 \theta$ (c) $\sin^2 \theta$ (d) $4 \sin^2 \theta$

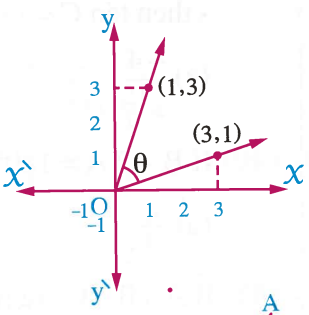
(49) If $\frac{\sin 3\theta}{\sin \theta} + \frac{\cos 3\theta}{\cos \theta} = 2$, then $\cos 2\theta =$ .....

(a) $\frac{-\sqrt{3}}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

(50) S.S. of the equation $\cos \chi + \cos 2\chi =$ zero where $\chi \in]0 , \pi]$ is .....

(a) $\{\pi\}$ (b) $\{\frac{\pi}{6} , \pi\}$ (c) $\{\frac{\pi}{3}\}$ (d) $\{\frac{\pi}{3} , \pi\}$

(51) In the opposite figure :

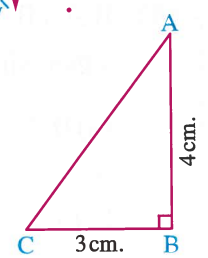


$\cot \theta =$ .....

(a) $\frac{4}{5}$ (b) $\frac{3}{4}$

(c) $\frac{2}{3}$ (d) $\frac{3}{5}$

(52) In the opposite figure :

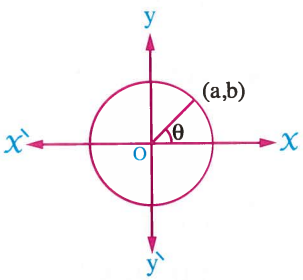


$\sin A + \sin 2C + \sin 3B =$ .....

(a) $\frac{14}{25}$ (b) $\frac{13}{25}$

(c) $\frac{11}{12}$ (d) $\frac{16}{25}$

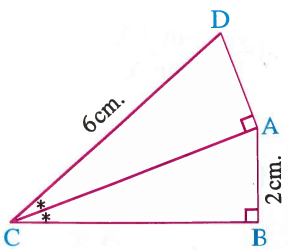
(53) The opposite figure represents the unit circle , then $\tan \frac{\theta}{2} =$ .....



(a) $\frac{b}{2a}$ (b) $\frac{b}{1+a}$

(c) $\frac{2b}{a}$ (d) $\frac{1+b}{1+a}$

(54) In the opposite figure :

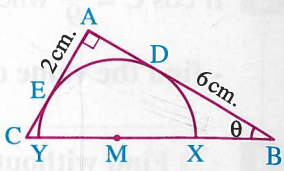


$\sin (\angle DCB) =$ .....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(55) In the opposite figure :

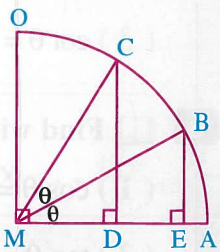


$\tan 2\theta =$ .....

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

(56) In the opposite figure :

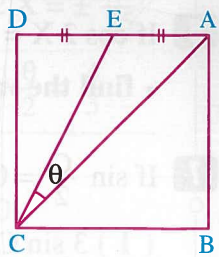


A quarter of a circle M , BE = 3 cm. , CD = 5 cm. , then $\cos\theta =$ .....

(a) $\frac{5}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

(57) In the opposite figure :



ABCD is a square , then $\cot 2\theta =$ .....

(a) $\frac{5}{2}$ (b) 2

(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

Second Essay questions

1 Without using the calculator , find the value of each of the following :

(1) 2 sin 22° 30̀ cos 22° 30̀ «$\frac{1}{\sqrt{2}}$»

(2) $2\cos^2 15° - 1$ «$\frac{\sqrt{3}}{2}$»

(3) $\cos^2 67.5° - \sin^2 67.5$ «$\frac{-1}{\sqrt{2}}$»

(4) 4 sin 7.5° cos 7.5° cos 15° «$\frac{1}{2}$»

(5) $\frac{2\tan 22° 30̀}{1 - \tan^2 22° 30̀}$ «1»

(6) $\frac{2\tan\frac{\pi}{6}}{1 - \tan^2\frac{\pi}{6}}$ «$\sqrt{3}$»

(7) $\frac{\cos^2 23° - \sin^2 23°}{2\sin^2 23° - 1}$ «−1»

(8) $\frac{3}{\sec 15° \csc 15°}$ «$\frac{3}{4}$»

(9) $\frac{\cos 40° \cos 10° + \sin 40° \sin 10°}{\sin 15° \cos 15°}$ «$2\sqrt{3}$»

(10) $\frac{2\cos^2 165° - 1}{\sin 75° \cos 75°}$ «$2\sqrt{3}$»

2 If $\sin A = \frac{-3}{5}$ where $\pi < A < \frac{3\pi}{2}$, **find the value of each of :** sin 2 A , cos 2 A and tan 2 A «$\frac{24}{25}$, $\frac{7}{25}$, $\frac{24}{7}$»

3 If $\cos C = \frac{7}{9}$ where $C \in]0 , \frac{\pi}{2}[$

, **find the value of each of :** $\sin \frac{C}{2}$, $\cos \frac{C}{2}$ and $\tan \frac{C}{2}$ « $\frac{1}{3}$, $\frac{2\sqrt{2}}{3}$, $\frac{\sqrt{2}}{4}$ »

4 **Find without using the calculator the values of : $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ if :**

(1) $\sin \theta = \frac{4}{5}$, $0° < \theta < 90°$ | (2) $\cos \theta = \frac{1}{3}$, $0° < \theta < \frac{\pi}{2}$

(3) $\cot \theta = \frac{3}{2}$, $180° < \theta < 270°$ | (4) $\sin \theta = \frac{-12}{13}$, $\frac{3\pi}{2} < \theta < 2\pi$

5 **Find without using the calculator the values of : $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$:**

(1) $\cos \theta = \frac{1}{4}$, $0° < \theta < 90°$ | (2) $\sin \theta = \frac{3}{5}$, $90° < \theta < 180°$

(3) $\tan \theta = \frac{4}{3}$, $\pi < \theta < \frac{3\pi}{2}$ | (4) $\cos \theta = \frac{5}{13}$, θ is the measure of an acute angle.

6 If $\cos 2X = \frac{1}{8}$

, **find the value of each of :** $\sin X$ and $\cos X$ « $\pm \frac{\sqrt{7}}{4}$, $\pm \frac{3}{4}$ »

7 If $\sin \frac{C}{2} = 0.6$ where $C \in]0 , \frac{\pi}{2}[$, **find the value of :**

(1) $3 \sin C + 4 \cos C$ (2) $\sec C + \tan C$ « 4 , 7 »

8 If $\sin \frac{A}{2} = \frac{1}{3}$, without using the calculator

, **find the value of :** $\cos \frac{A}{2}$, $\sin A$, $\cos A$ « $\pm \frac{2\sqrt{2}}{3}$, $\pm \frac{4\sqrt{2}}{9}$, $\frac{7}{9}$ »

9 If A is the measure of an acute angle and $\cos 2A = \frac{119}{169}$, without using the calculator

, **find the value of each of :** $\sin A$, $\cos A$, $\tan 2A$ « $\frac{5}{13}$, $\frac{12}{13}$, $\frac{120}{119}$ »

10 If $\cos^2 A = \frac{25}{169}$ where $A \in]\pi , \frac{3\pi}{2}[$

Find the value of each of : $\sin 2A$ and $\cos \frac{A}{2}$ « $\frac{120}{169}$, $\frac{-2}{\sqrt{13}}$ »

11 If $25 \cos A + 7 = 0$ where A is the smallest positive angle , $4 \tan B - 3 = 0$ where $180° < B < 270°$

Find the value of each of :

(1) $\cos 2A$ (2) $\tan (A - 2B)$ « $\frac{-527}{625}$, $\frac{336}{527}$ »

12 If $\tan A = \frac{1}{3}$ where $A \in]0 , \frac{\pi}{2}[$ and $\tan B = \frac{1}{7}$ where $B \in]0 , \frac{\pi}{2}[$

Find the value of each of : $\tan (2A + B)$, $\tan (A - 2B)$ and $\tan 2(A + B)$ « 1 , $\frac{3}{79}$, $\frac{4}{3}$ »

13 If $\sin(270° + A) = \frac{5}{13}$, $\tan B = \frac{4}{3}$, $\pi < B < A < 2\pi$

Find the value of :

(1) $\cos(\pi + 2A)$ (2) $\cot\left(\frac{\pi}{2} - 2B\right)$ « $\frac{119}{169}$, $\frac{-24}{7}$ »

14 If $\sin 2X = \frac{120}{169}$ where $2X \in]0, 2\pi[$

, **find without using the calculator the value of :** $\sin X + \cos X$ « $\frac{17}{13}$ »

15 **If** $\sin 2X = \frac{1}{5}$ **Find the value of :**

(1) $\sin 3X \cos X + \cos 3X \sin X$ (2) $\cos 5X \cos X + \sin 5X \sin X$ « $\pm\frac{4\sqrt{6}}{25}$, $\frac{23}{25}$ »

16 **If** $\sin X + \cos X = \frac{7}{5}$ **where** $0 < X < \frac{\pi}{2}$ **, then prove that :**

(1) $\sin 2X = \frac{24}{25}$ (2) $\sec X + \csc X = \frac{35}{12}$ (3) $\cos 2X = \pm\frac{7}{25}$

17 🕮 If $4\sin\theta + 3\cos\theta = 3$, prove without using the calculator that : $\tan\frac{\theta}{2} = \frac{4}{3}$, where $\frac{\theta}{2}$ is the measurement of a positive acute angle.

18 ABC is an acute-angled triangle in which $\tan A = 3$ and $4 + 3\tan 2B = 0$

Without using the calculator calculate : $\tan B$ and $m(\angle C)$ « 2 , 45° »

19 If $\tan A = \frac{1}{2}$ and $\tan B = 7$, then find $\tan 2A$ and hence

Prove that : $2A + B = 135°$ where A and $B \in]0, \frac{\pi}{2}[$ « $\frac{4}{3}$ »

20 If $\tan 2A = \frac{3}{4}$ where $A \in]0, \frac{\pi}{2}[$ and $\tan B = \frac{3}{2}\tan A$ where $B \in]0, \frac{\pi}{2}[$

Prove that : $A + B = 45°$

21 **Without using the calculator , and by using the trigonometric function of half the angle , find the value of each of :**

(1) 🕮 $\cos 75°$ (2) 🕮 $\tan 15°$

(3) 🕮 $\cos 22° 30'$ (4) $\tan 67° 30'$

22 🕮 **Geometry :** X Y Z is a triangle in which $x = 12$ cm. , $y = 18$ cm. , $z = 15$ cm. , **prove that :** $m(\angle Y) = 2\,m(\angle X)$

23 ABC is a triangle in which $a : b : c = 3 : 4 : 5$

Find the value of each of : $\sin A$ and $\tan 2A$ « $\frac{3}{5}$, $\frac{24}{7}$ »

24 ABC is a triangle in which $a = c$ **Prove that :** $\sin B = 2\sin A \cos C$

25 **Without using the calculator , prove that :**

(1) $\frac{\cos 2A}{\cos A - \sin A} = \cos A + \sin A$

(2) $\sin 8A = 8 \sin A \cos A \cos 2A \cos 4A$

(3) $2 \sin^2 2A + \cos 4A = 1$

(4) $1 - 2 \sin^2 \left(\frac{\pi}{4} - \frac{5X}{2} \right) = \sin 5X$

(5) $\cos^4 X - \sin^4 X = \cos 2X$

(6) $\frac{\sin A + \cos 2A}{1 + 2 \sin A} = 1 - \sin A$

(7) $\sin A = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$

(8) $\tan X = \frac{\sin X - 2 \sin^3 X}{2 \cos^3 X - \cos X}$

(9) $\frac{\sin 2X}{1 + \cos 2X} = \tan X$

(10) $\sec 2X = \frac{\sec^2 X}{1 - \tan^2 X}$

(11) $\frac{\sin 2X + \sin X}{1 + \cos X + \cos 2X} = \tan X$

(12) $\frac{1 - \cos C + \sin C}{1 + \cos C + \sin C} = \tan \frac{C}{2}$

(13) $\frac{\cos^2 A}{\cot \frac{A}{2} - \tan \frac{A}{2}} = \frac{1}{4} \sin 2A$

(14) $\frac{1 - \sin 2X}{\cos 2X} = \frac{1 - \tan X}{1 + \tan X}$

(15) $\frac{1 - \cos \theta}{1 + \cos \theta} = \tan^2 \frac{\theta}{2}$

(16) $4 \sin A \cos^3 A - \sin 2A = \frac{1}{2} \sin 4A$

(17) $\frac{1 - \cos 2A}{3 + \cos 2A} = \frac{\sin^2 A}{1 + \cos^2 A}$

(18) $\frac{\cot \theta - 1}{\cot \theta + 1} = \frac{\cos 2\theta}{1 + \sin 2\theta}$

(19) $(\sin X + \cos X)^2 - (\sin X - \cos X)^2 = 2 \sin 2X$

(20) $2 \cos (45° - 2A) \cos (45° + 2A) = 1 - 8 \sin^2 A \cos^2 A$

(21) $(\sin A - \sin B)^2 + (\cos A + \cos B)^2 = 4 \cos^2 \left(\frac{A + B}{2} \right)$

(22) $\cos^2 A = \frac{1}{2} (1 + \cos 2A)$ and hence deduce the value of $\cos 15°$ « $\frac{\sqrt{6} + \sqrt{2}}{4}$ »

(23) $\csc 2X + \cot 2X = \cot X$ and hence deduce the value of $\cot 15°$ « $2 + \sqrt{3}$ »

(24) $\cos 2X = \frac{1 - \tan^2 X}{1 + \tan^2 X}$ and hence deduce the value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ « $\frac{\sqrt{3}}{2}$ »

---

26 **Without using the calculator , prove that :**

(1) $16 \cos 20° \cos 40° \cos 60° \cos 80° = 1$

(2) $\sin \frac{5\pi}{24} \cos^3 \frac{5\pi}{24} - \sin^3 \frac{5\pi}{24} \cos \frac{5\pi}{24} = \frac{1}{8}$

(3) $\tan^2 15° + 2\sqrt{3} \tan 15° = 1$

27 **Find the solution set of each of the following equations where $X \in]0 , 2\pi[$:**

(1) 📖 $\sin 2X = \sin X$ « $\{\frac{\pi}{3} , \pi , \frac{5\pi}{3}\}$ »

(2) $\sin 2X = \sqrt{3} \sin X$ « $\{\frac{\pi}{6} , \pi , \frac{11\pi}{6}\}$ »

(3) 📖 $\cos 2X + \sin X = 0$ « $\{\frac{7\pi}{6} , \frac{11\pi}{6} , \frac{\pi}{2}\}$ »

(4) $\sqrt{3} \cos 2X = \cos X$ « $\{30° , 330° , 125° 16\grave{} , 234° 44\grave{}\}$ »

(5) $\cos 2X + 3 \cos X + 2 = 0$ « $\{\pi , \frac{2\pi}{3} , \frac{4\pi}{3}\}$ »

(6) $\sin^2 2X + 4 \sin X \cos X + 1 = 0$ « $\{\frac{3\pi}{4} , \frac{7\pi}{4}\}$ »

(7) 📖 $\cos^2 X - \sin^2 X = -\frac{1}{2}$ « $\{\frac{\pi}{3} , \frac{2\pi}{3} , \frac{4\pi}{3} , \frac{5\pi}{3}\}$ »

(8) 📖 $\tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} = 1$ « $\{\frac{\pi}{4} , \frac{5\pi}{4}\}$ »

(9) 📖 $\cos X - 2 \sin^2 \frac{1}{2} X = 0$ « $\{\frac{\pi}{3} , \frac{5\pi}{3}\}$ »

---

28 **Find the solution set of the equation :** $\sin X \cos^3 X - \sin^3 X \cos X = \frac{\sqrt{3}}{8}$, $X \in]0 , \pi[$

« $\{\frac{\pi}{12} , \frac{\pi}{6} , \frac{7\pi}{12} , \frac{2\pi}{3}\}$ »

---

29 If $5 \cos X + 4 = 0$, $X \in]0 , \pi[$ **Find the value of :** $\cos 3X$ « $\frac{44}{125}$ »

---

30 **If ABC is a right-angled triangle at C , prove that :**

(1) $\sin 2A = \frac{2ab}{c^2}$ (2) $\cos 2A = \frac{b^2 - a^2}{c^2}$ (3) $\sin \frac{A}{2} = \sqrt{\frac{c - b}{2c}}$

---

31 ABC is a triangle , $\overrightarrow{AH}$ bisects the interior angle A such that : $\overrightarrow{AH} \cap \overline{BC} = \{H\}$

Prove that : $AH = \frac{2bc}{b + c} \cos \frac{A}{2}$

---

32 📖 **Mechanics :** A football player kicked the ball by an angle of measurement 30° with the ground surface with initial velocity of magnitude of 14.7 m./sec.

If the horizontal distance S which the ball covers is given by the relation :

$S = \frac{2 v_o^2 \sin \theta \cos \theta}{g}$ where g is the free fall acceeration and equals 9.8 m./sec.^{2}

and V_o represents the initial velocity.

(1) Put the previous relation in the simplest form.

(2) Find the horizontal distance S which the ball covers in meters. « 19 m. »

Third Higher skills

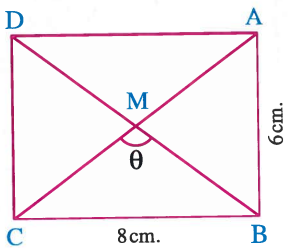
1 Choose the correct answer from those given :

(1) If $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A} = 2$, then $\cos 2A =$ .....

(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

(2) In the opposite figure :

If ABCD is a rectangle in which AB = 6 cm. , BC = 8 cm. , then $\sin\theta =$ .....



(a) $\frac{24}{5}$ (b) $\frac{24}{25}$

(c) $\frac{4}{5}$ (d) $\frac{12}{25}$

(3) The solution set of the equation : $\cos 3X - \cos 2X + \cos X = 0$, where X is an acute angle is .....

(a) $\{30°, 60°\}$ (b) $\{30°, 45°\}$ (c) $\{45°, 60°\}$ (d) $\{15°, 75°\}$

(4) If $3A + 2B = 180°$, then $\frac{\sin 2A \cos B - \cos 2A \sin B}{\sin A \cos 3B + \cos A \sin 3B} =$ .....

(a) – 1 (b) zero (c) 1 (d) 2

(5) If $0 < \theta < \frac{\pi}{4}$, then $\sqrt{1+\sin 2\theta} - \sqrt{1-\sin 2\theta} =$ .....

(a) $\sin\theta$ (b) $\cos\theta$ (c) $2\sin\theta$ (d) $2\cos\theta$

---

2 Without using the calculator , prove that :

(1) $\sin 54° \sin 18° = \frac{1}{4}$ **(2)** $\sin 54° - \sin 18° = \frac{1}{2}$

---

3 Without using the calculator , **prove that :** $\tan 20° \tan 30° \tan 40° = \tan 10°$

---

4 Prove that : $\tan 3X = \frac{3\tan X - \tan^3 X}{1 - 3\tan^2 X}$

, **then prove that :** $\tan 50° \tan 60° \tan 70° = \tan 80°$

---

5 In the triangle ABC , $b = 2\sqrt{3}\, a$ and $m(\angle C) = 30°$

Without using the calculator prove that : $\tan A = \frac{\sqrt{3}}{9}$

Exercise

19

Heron's formula

From the school book | Understand | Apply | Higher Order Thinking Skills | Test yourself

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The surface area of the triangle whose two side lengths are 6 cm. and 8 cm. and the measurement of the angle included between them is 30° equals ..... cm^2.

(a) 10 (b) 12 (c) 14 (d) 24

(2) The area of ΔABC in which AB = 7 cm. , BC = 8 cm. and m ($\angle B$) = 50° equals ..... (approximated to two decimals)

(a) 21.45 (b) 45.21 (c) 7.56 (d) 8.45

(3) ABC is an equilateral triangle of area $36\sqrt{3}$ cm^2 , then its side length equals ..... cm.

(a) 6 (b) $6\sqrt{3}$ (c) 12 (d) $12\sqrt{3}$

(4) The surface area of the isosceles triangle which one of its equal side lengths is 10 cm. and the measurement of one of its base angles is 45° equals ..... cm^2

(a) 25 (b) 50 (c) 100 (d) 200

(5) The area of the rohmbus in which the measure of one of its angles is 50° and its side length is 6 cm. is ..... cm^2 (approximated to the nearest cm^2)

(a) 20 (b) 24 (c) 28 (d) 32

(6) The area of the equilateral triangle whose side length is x cm. equals ..... cm^2

(a) x^2 (b) $\frac{\sqrt{3}}{2}x^2$ (c) $\frac{\sqrt{3}}{4}x^2$ (d) $\frac{1}{2}x^2$

(7) If the perimeter of a triangle is 60 cm. and the length of one side is 26 cm. , then the lengths of the other two sides in centimeters can be .....

(a) 4 , 30 (b) 31 , 3 (c) 20 , 14 (d) 2 , 32

(8) Area of Δ ABC in which P – a = 3 cm. , P – b = 5 cm. , P – c = 9 cm. where P is half the perimeter of the triangle equals ..... cm^2

(a) $3\sqrt{7}$ (b) $4\sqrt{215}$ (c) $3\sqrt{255}$ (d) 46

(9) The surface area of the triangle whose side lengths are 6 cm. , 8 cm. and 10 cm. is ..... cm^2

(a) 24 (b) 30 (c) 40 (d) 48

(10) Area of triangle ABC in which AB = AC = 6 cm. , $\tan A = \frac{24}{7}$ equals ..... cm^2

(a) 61.71 (b) 34.56 (c) 17.28 (d) 5.04

(11) Area of triangle whose side lengths are 24 cm. , 36 cm. , 60 cm. equals ..... cm^2

(a) 60 (b) zero (c) 48 (d) 72

(12) A triangle whose perimeter 590 cm. and ratio among its side lengths is 14 : 20 : 25 , then its area ≃ ..... cm^2

(a) 13981 (b) 14182 (c) 14181 (d) 13982

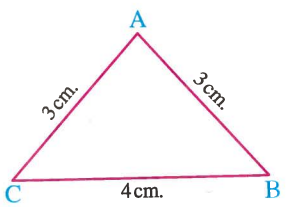
(13) Ratio among side lengths of a triangle is 7 : 5 : 3 and its area 2598 cm^2 , then perimeter of the triangle ≃ ..... cm.

(a) 100 (b) 200 (c) 300 (d) 400

(14) If the side lengths of a triangle are 35 cm. , 54 cm. , 61 cm. , then its longest height = ... cm.

(a) 16 (b) $24\sqrt{5}$ (c) 28 (d) $40\sqrt{2}$

(15) **In the opposite figure :**



The surface area of Δ ABC equals ..... cm^2

(a) 20 (b) $4\sqrt{5}$

(c) $2\sqrt{5}$ (d) 10

(16) The area of an isosceles triangle if the length of one of its equal sides is 14 cm. and its perimeter is 36 cm. equals ..... cm^2

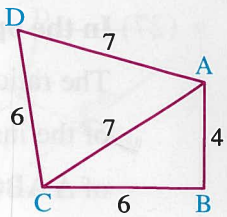
(a) $12\sqrt{5}$ (b) $18\sqrt{5}$ (c) $24\sqrt{5}$ (d) $30\sqrt{5}$

(17) Area of quadrilateral ABCD in which m (∠ B) = 90° , AB = 5 cm. , BC = 12 cm. , AD = CD = 13 cm. ≃ ..... (to nearest cm^2)

(a) 104 (b) 103 (c) 98 (d) 105

(18) In the opposite figure :

The area of the quadrilateral to the nearest cm^2 is .....

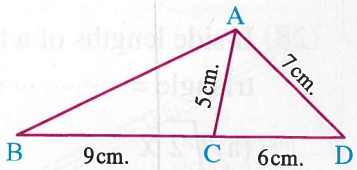


(a) 31 (b) 39

(c) 45 (d) 80

(19) In the opposite figure :

The area of (Δ ABC) = ... cm^2

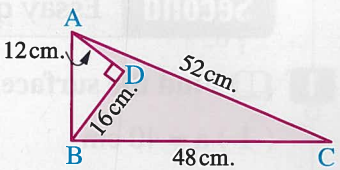


(a) $6\sqrt{6}$ (b) $9\sqrt{6}$

(c) $9\sqrt{3}$ (d) $2\sqrt{6}$

(20) In the opposite figure :

Area of the shaded part = ... cm^2



(a) 96 (b) 384

(c) 480 (d) 576

(21) If (Δ) is the area of a triangle and (2 P) is the perimeter of the same triangle , then the diameter length of the inscribed circle of the triangle equals .....

(a) $\frac{\Delta}{P}$ (b) $\frac{\Delta}{2P}$ (c) $\frac{2\Delta}{P}$ (d) $\frac{3\Delta}{2P}$

(22) Which of the following equals the length of the radius of the circle which touches the sides of the Δ ABC internally ?

(a) $\frac{a}{2 \sin A}$ (b) $\frac{a+b+c}{\sin A + \sin B + \sin C}$

(c) $\frac{a\, b \sin C}{a+b+c}$ (d) $\frac{1}{2}\, a\, c \sin B$

(23) Radius of the circle that touches the sides of Δ ABC internally where a = 25 cm. , b = 17 cm. , c = 26 cm. equals ..... cm.

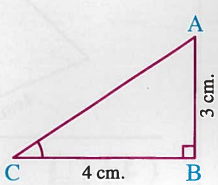
(a) 8 (b) 10 (c) 7 (d) 6

(24) If the perimeter of a triangle = 25 cm. , the radius length of the circle touches its sides internally = 4 cm. , then its area = ... cm^2

(a) 4 π (b) 25 (c) 50 (d) 75

(25) In the opposite figure :

The sum of the lengths of the two radii of the inscribed circle and circumcircle of the Δ ABC equals ..... cm.



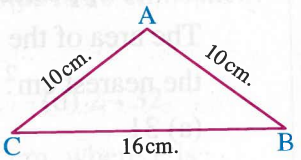
(a) 4 (b) 3 (c) 5 (d) 3.5

(26) The ratio between the lengths of the two radii of inscribed circle and the circumcircle of an equilateral triangle = ...

(a) 1 : 4 (b) 1 : 3 (c) $1 : \sqrt{3}$ (d) 1 : 2

(27) In the opposite figure :

The ratio between the radius lengths of the inscribed circle and the circumcircle of ΔABC = ...



(a) 3 : 8 (b) 8 : 25 (c) 3 : 25 (d) 3 : 4

(28) If side lengths of a triangle are x , $x+2$, $2x-2$ where $x \geq 3$, then area of the triangle = ... square unit.

(a) $\sqrt{2x}$ (b) $x\sqrt{x-2}$ (c) $3x\sqrt{x-2}$ (d) $2x\sqrt{x-2}$

Second Essay questions

1 Find the surface area of ΔABC , if possible , in each of the following cases :

(1) a = 40 cm. , b = 24 cm. and c = 32 cm. « 384 cm^2 »

(2) b = 16 cm. , c = 20 cm. and m ($\angle A$) = 60° « $80\sqrt{3}$ cm^2 »

(3) a = 5 cm. , b = 12 cm. and c = 13 cm. « 30 cm^2 »

(4) Its side lengths are 12 cm. , 14 cm. and 30 cm.

2 Find the radius length of the inscribed circle of ΔABC in each of the following cases :

(1) a = 7 cm. , b = 9 cm. and c = 14 cm. « $\frac{4\sqrt{5}}{5}$ cm. »

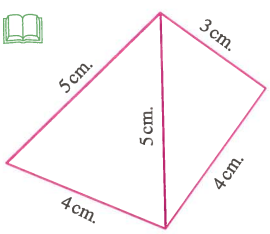
(2) a = 5 cm. , b = 6 cm. and m ($\angle C$) = 60° « 1.6 cm. »

3 **Yards :** A triangular yard at which the ratio among its side lengths is 7 : 5 : 3

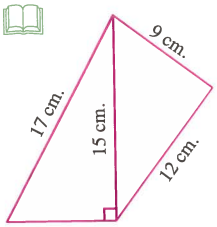
If the perimeter of the yard is 300 metres , find its surface area. « 2598 m^2 »

4 Find the surface area of each of the following figures using the given data shown :

(1)



(2)



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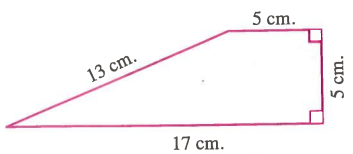

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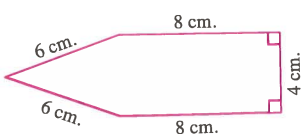
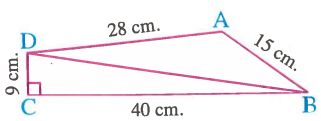
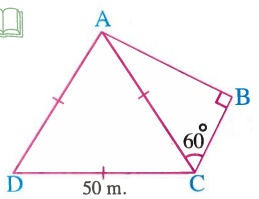


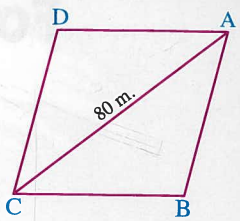
(5)



(6)



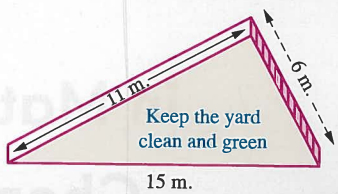
5 The opposite figure represents a rhombus-like piece of land whose perimeter is 200 metres and length of $\overline{AC}$ = 80 metres. Find its surface area.



« 2400 m^2. »

6 **Environment :**

The following diagram illustrates a triangular yard. Find its surface area.



« $20\sqrt{2}$ cm^2. »

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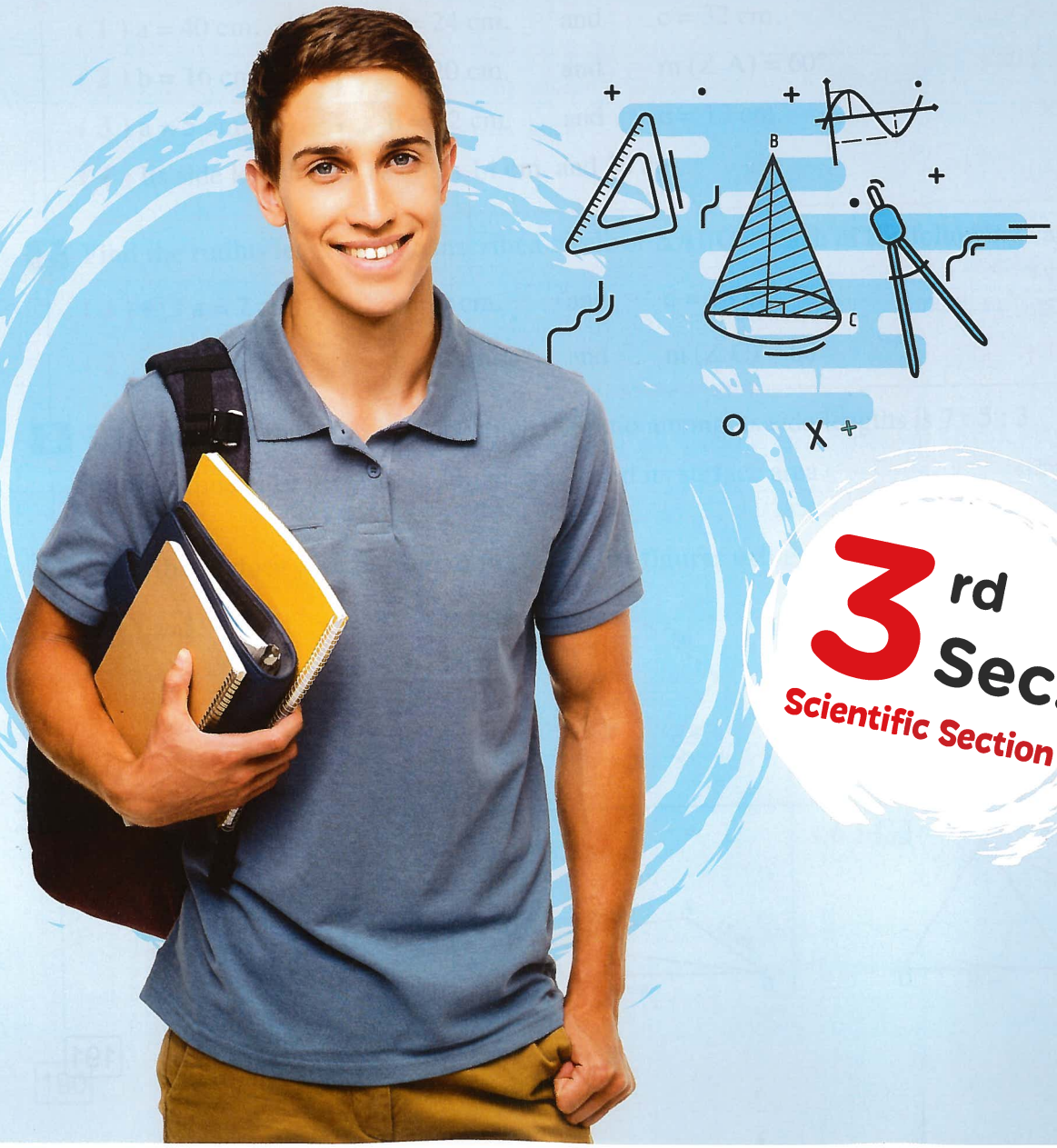

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